VOLUME LXIV NUMBER 3

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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PLATE I

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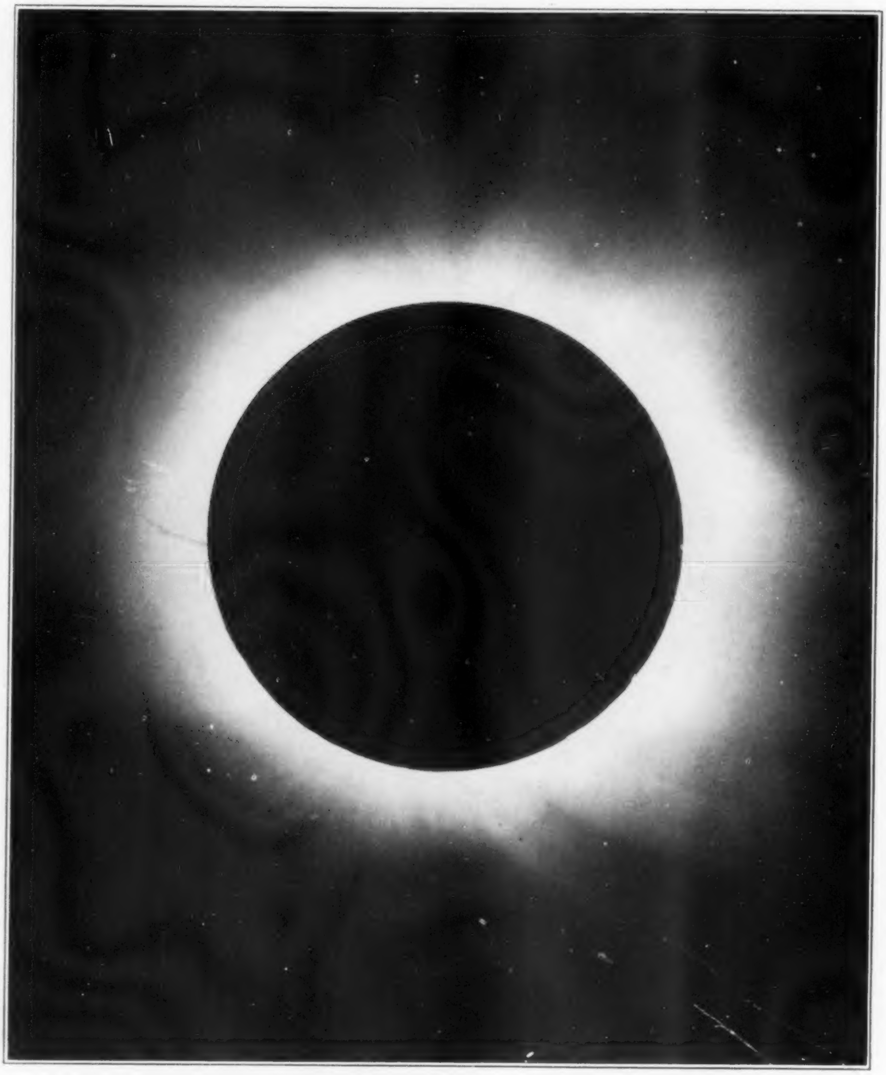

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THE ECLIPSE OF JANUARY 24, 1925, TAKEN WITH A 4-INCH LENS OF 28 FEET FOCUS

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIV OCTOBER 1926 NUMBER 3

THE 1925 CORONA

By FREDERICK SLOCUM

ABSTRACT

The path of totality of the eclipse of the sun of January 24, 1925, passed directly over the Van Vleck Observatory. The duration of totality at that point was 112 seconds.

Photographs of the *corona* and prominences were made with the 20-inch visual refractor and other instruments. *Photographs* and *a sketch* from the negatives are reproduced.

The corona is of an *intermediate* type. The *equatorial wings and polar rays are characteristic of the minimum sun-spot coronas,* but there is a perfect coronal *dome,* terminating in a long spike, which is a *typical feature of a maximum* of sun-spots.

A preliminary report of the observations made at the Van Vleck Observatory of the total eclipse of the sun on January 24, 1925, was given in *Popular Astronomy*, **33**, 169, 1925. I propose here to give a more detailed description of the corona.

Clouds covered the sky during the early morning, but cleared away just before the beginning of totality so that successful photographs of the corona were secured.

For this purpose nine different instruments were used as follows: the 20-inch visual refractor of 28 feet focal length, a 4-inch of 25 feet focus, a 3-inch of 12 feet focus (loaned by Brown University), a 6-inch of 7 feet focus, two 5-inch portrait doublets (loaned by Yale), a short-focus 4-inch doublet, a 2-inch Balopticon lens, and a 1-inch landscape camera. The last two were mounted on fixed supports; all the others were attached to the 20-inch equatorial mounting as shown in Plate II.

The three doublets were exposed for 100 seconds with Eastman

40 plates. On account of fog by sky light and solarization of the inner corona these plates are of little value.

The scale of the fixed cameras is so small that their plates give only the general outline and extent of the corona and its orientation with respect to the horizon (see Plate III*a*). These plates show a sharp horn in the northwest quadrant extending out nearly two diameters of the sun, and the equatorial wings which are characteristic of the minimum type of corona. The western wing is longer and wider than the eastern, resembling, in this respect, the coronas of 1878 and 1900, and being the reverse of the coronas of 1868, 1889, and 1914. This is in agreement with what has been noted before, viz., that the wings are, in general, not symmetrical and that the "fish tail" of the corona changes its direction at each sun-spot minimum.

The 20-inch refractor was used as for the routine stellar parallax work with "minus blue" filter and Cramer Instantaneous Isochromatic plates. The filter is $4\frac{3}{8}$ inches wide and the diameter of the moon's image was $3\frac{1}{4}$ inches, so that only the inner portions of the corona appear on the plates. The 6-inch was also used with filter and Isochromatic plate (see Plate III*b*).

So far as I can see, the yellow-sensitive plates show the same coronal features as the blue-sensitive plates, but the details are not so sharp. The prominences on the yellow plates are very weak, and this is to be expected because only the helium light of the prominences is utilized, whereas, on the blue plates, calcium and blue hydrogen lights are effective.

The best coronal pictures were obtained with the long-focus 4- and 3-inch lenses. The former is a permanent fixture of the 20-inch equatorial, and is used as a long focus finder or guiding telescope. Normally it has no tube, but one of black cloth was provided for the eclipse. It is a visual lens, but on account of the large ratio of aperture to focal length, 1:75, it was found possible to use it to good advantage without a filter. The 3-inch is a photographic lens with ratio 1:48.

For results obtained with these two lenses, see Plates I, V, and IV. On account of the well-known difficulty of reproducing eclipse photographs, I have made a rough drawing from the nega-

PLATE II

THE 20-INCH REFRACTOR OF THE VAN VLECK OBSERVATORY WITH SIX CAMERAS ATTACHED AS USED FOR THE 1925 ECLIPSE

PLATE III

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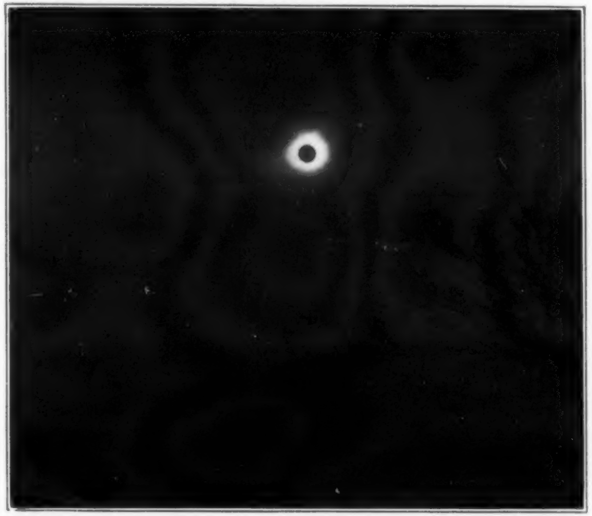

a) The Eclipse of January 24, 1925, Taken with a 1-inch Landscape Camera

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b) The Eclipse of January 24, 1925, Taken with a 6-inch Visual Refractor and Yellow Filter

tives (Plate VI). No attempt has been made to represent the gradations of light correctly, but most of the lines have been carefully traced from the originals. All the details mentioned in the following description may be identified on the drawing.

The axis of the sun is marked by two short lines within the circumference of the moon. The north pole of the sun's rotation was 9° west of terrestrial north. The polar rays are very conspicuous at both poles. At the north pole they cover an arc of 100° and at the south pole, of 65°, measurements being made to the edge of the equatorial wings. The points of divergence are well marked. For want of a better name, I shall call these the "magnetic poles." The north magnetic pole is 7° east of the sun's north pole of rotation, and the south magnetic pole is 10° east of the south pole of rotation. This asymmetry has frequently been noticed before. The asymmetry with respect to the wings is even more marked, the north magnetic pole being 38° from the east wing and 62° from the west, while the south magnetic pole is 27° from the east wing and 38° from the west wing. The rays, in places, seem like fine, sharp lines, having the appearance of "combed hair," and in other places resemble bands or bundles of lines. This is well shown west of the north pole. At 13° east there is what may be a rift or gap, but on the negatives it looks quite different from the sky background, and I am inclined to think that it may be a dark ray.

The wings show comparatively little detail, as is typical of a minimum corona. The edges on both sides show graceful reversed curves. Just within the edges of the eastern wing appear two horns, slightly brighter than the rest of the wing. The southern one is directly over the largest and brightest prominence on the eastern limb. Near the center of this wing are two parallel streamers at equal distances from the sun's equator, both starting from a point near, but not exactly at, small prominences.

Near the southern edge of the west wing is a narrow leaflike formation, its western edge resting upon a small but bright prominence. A few degrees farther west is a small prominence covered by two faint concentric hoods. Near the middle of the base of the west wing is the largest prominence on the sun's limb. It extends from

latitude 5° N. to 17° S., and rises to a height of 40,000 miles (64,000 km). I can see no particular disturbance of the corona in the vicinity of this prominence.

Between the south pole and the west wing is a curious tangential formation. It gives the impression of a mighty current which has cut sharply off most, but not all, of the polar rays along 30° of the sun's limb. Some of the rays may be seen projecting above the straight tangent, but whether they are in front of the tangent or behind it I cannot tell. A somewhat similar formation was observed by Khandrikoff during the 1887 eclipse.

By far the most striking feature of the 1925 corona is the series of arches or domes in the northwest quadrant, extending from latitude 40° to 70° N., or a distance of about 225,000 miles along the limb and rising to a height of 400,000 miles. Such forms are common in coronas observed near the time of sun-spot maximum, and this one, combined with the characteristics of minimum type, apparently marks the 1925 corona as of an intermediate type. Synclinal, leaf-like, petal-shaped, "arches" and "cones" are some of the terms used to characterize these formations. To me the word "domes" seems to convey the best idea. In this case there are four domes almost concentric, the inner three having rounded tops, the outer one terminating in a point, from which extends a perfectly straight rodlike form which can be traced to a distance of 1,500,000 miles on some of the plates.

The connection between these domes and solar prominences has frequently been mentioned. There are several small prominences within this dome, one of them quite bright, but not at the center. Whether there was a disturbed area just below the limb, I do not know.

VAN VLECK OBSERVATORY, MIDDLETOWN, CONN.
July 1, 1926

PLATE IV

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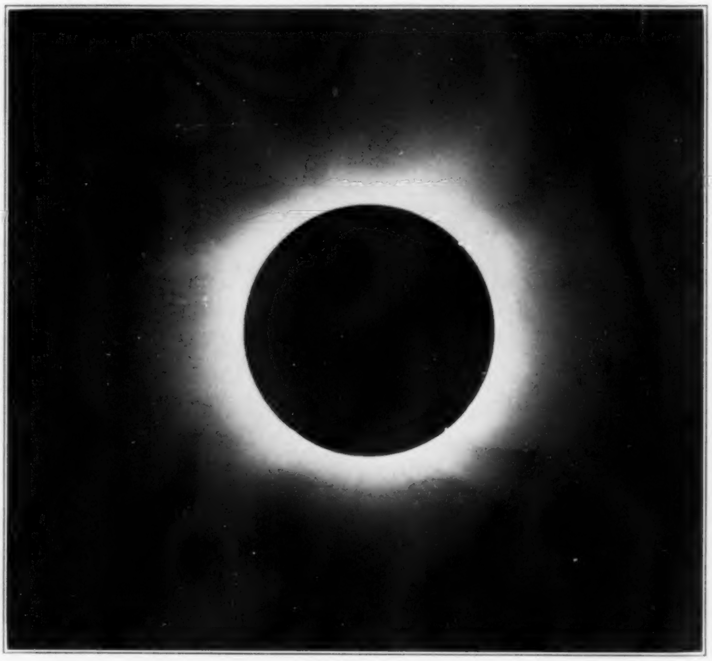

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THE ECLIPSE OF JANUARY 24, 1925, TAKEN WITH A 3-INCH LENS OF 25 FEET FOCUS

PLATE V

PLATE V

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INNER CORONA AND PROMINENCES JANUARY 24, 1925, WITH A 4-INCH LENS OF 28 FEET FOCUS

PLATE VI

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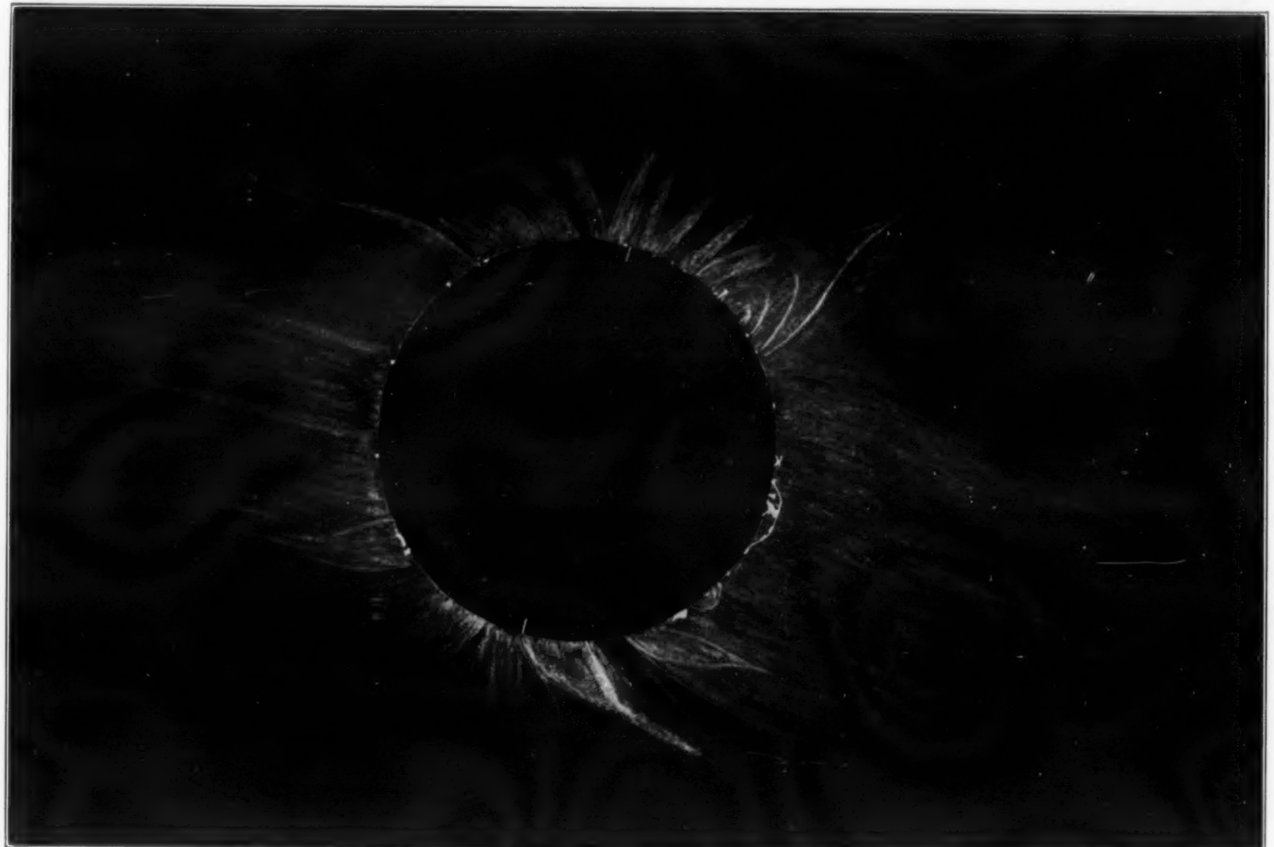

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THE ECLIPSE OF JANUARY 24, 1925, DRAWN FROM THE NEGATIVES BY F. SLOCUM

REMARKS ON VARIOUS STATISTICAL PROPERTIES OF GALACTIC CEPHEIDS HAVING PERIODS LONGER THAN ONE DAY[1]

By J. SCHILT[2]

ABSTRACT

Maximum frequency of Cepheids in different galactic longitudes.—In the region of Sagittarius and Aquila the frequency-curve shows a maximum for log $P=0.84$. The maximum for other regions of the galaxy is for a decidedly shorter period. The period of maximum frequency for the Cepheids in the Small Magellanic Cloud is about the same as for the galactic Cepheids in the hemisphere opposite to the Sagittarius-Aquila region.

Relation between magnitude, proper motion, and period.—The present material shows that the median apparent magnitude, median proper motion, and median absolute magnitude are systematically small for log P between 0.8 and 1.0. There is no decrease of apparent magnitude and proper motion for periods longer than ten days, as would be expected from a uniform distribution in density and velocity.

Absolute magnitudes.—The fact that the absolute magnitudes derived from the motions of the fainter Cepheids differ appreciably from the generally adopted values based on thirteen Boss stars indicates the importance of obtaining both accurate proper motions and additional radial velocities for these stars.

The present investigation is based upon the variable stars listed in the "Katalog und Ephemeriden veränderlicher Sterne für 1926,"[3] having periods between one and eighty days, which are not known to be eclipsing binaries. The number of these stars is one hundred sixty-three; twenty-one additional stars having Cepheid characteristics are of unknown period. Three groups have been formed according to period as follows:

Group	Period in Days	Stars
I	1–6	46
II	6–12	51
III	12–80	66

The definition of a Cepheid is rather vague, especially for stars of unknown spectrum. Constancy of period is not a reliable criterion, since variation in the period is not uncommon for true Cepheids and may amount, as in the case of ζ Geminorum,[4] to 10 per cent of the

[1] *Contributions from the Mount Wilson Observatory*, No. 315.

[2] Fellow of The International Education Board.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **60**, 228, 1925.

[4] *See* W. Rabe, *Astronomische Nachrichten*, **219**, 129, 1923.

period. On the other hand, as observations accumulate, many stars will be found to have varying periods.

Although the spectral types have not always been given in the "Katalog," it is possible to make a rough division into spectral classes for the stars concerned:

Group	A8	G	K	M	Rp	Nb	Unknown
I	18	16	2	1	1	......	10
II	15	17	3	1	......	......	13
III	7	25	9	8	2	1	14

The rule that longer periods are correlated with more advanced spectral types seems to be confirmed by the frequencies of group III, as compared with those of groups I and II; but whereas the bulk of the stars in groups I and II are probably Cepheids, this is not certainly true for group III. The relative number of K and M stars in this group is larger than in the preceding groups, as is to be expected; but the group may include some Md variables, which are usually treated as a separate class, for it is difficult to decide where the longer-period Cepheids end and the class of long-period variables begins. For the present I have excluded all stars of known spectral type later than K and, moreover, the stars of group III of unknown spectral type. Two additional stars have also been excluded, viz., RU Doradus, which probably belongs to the Magellanic Cloud, and RX Camelopardalis, since the elements, $P=7.908$, $M-m=5.57$, give rise to doubt. The remaining stars for the three groups are forty-four, forty-eight, and forty-one in number, respectively.

The distribution in galactic longitude is given in Table I.

The mean of the galactic latitudes without regard to sign and the algebraical mean latitudes are also entered in the table. The difference in number for different galactic longitudes is for the most part spurious. The lack of Cepheids in longitudes 180° to 240°, or more accurately between the limits 173° and 230°, may, however, be real in part, since a considerable number of long-period and eclipsing variables are known in this region. The number of Cepheids found in different galactic longitudes is influenced by the study of special regions, but no such selective effect is to be expected in the distribu-

tion of periods among the different longitudes. Table I shows, however, that the number of stars of group I in the region 300° to 30° is very small as compared with those of groups II and III, the numbers being two, twenty, and twelve, respectively. It seems worth while, therefore, to investigate the distribution of periods more in detail.

The numbers of stars having different values of log P (P = period in days) have been entered in Table II for three intervals of galactic longitude, viz., 30°–150°, 150°–270°, and 270°–30°. Similar data for

TABLE I

DISTRIBUTION OF CEPHEIDS IN GALACTIC CO-ORDINATES

Gal. Long.	Group I			Group II			Group III			Total No.
	No.	Lat.*	Lat.†	No.	Lat.*	Lat.†	No.	Lat.*	Lat.†	
0°– 30°..	0	......	......	5	9°	−4°	5	10°	−7°	10
30 – 60 ..	2	6°	−4°	3	6	+1	5	6	0	10
60 – 90 ..	8	4	+4	3	1	−1	1	2	−2	12
90 –120 ..	6	8	+4	3	2	−2	3	4	0	12
120 –150 ..	2	11	−11	3	1	+1	3	10§	−10§	8
150 –180 ..	4	12	−6	4	12	+12	4	2	0	12
180 –210 ..	0	......	......	0	......	......	1	0	0	1
210 –240 ..	1	3	−3	0	......	......	1	1	+1	2
240 –270 ..	15	2	0	7	4	−1	10	4	+1	32
270 –300 ..	4	6	−3	5	14	−1	1	58	+58	10
300 –330 ..	0	......	......	4	10	−8	1	4	−4	5
330 –360 ..	2	4	−4	11	6‡	−6‡	6	6	+5	19

* Mean latitude without regard to sign.

† Algebraic mean latitude.

‡ Excluding RY Boötis, lat. +62°.

§ Excluding RS Ceti, lat. −52°.

the Small Magellanic Cloud, taken from *Harvard Circular*, No. 280, have been added for comparison.

The quantities $\Sigma\frac{1}{6}$ and $\Sigma\frac{1}{2}$, indicating the limits of log P below which lie $\frac{1}{6}$ and $\frac{1}{2}$, respectively, of the stars, have been determined graphically from smoothed, integrated frequency-curves. The mean errors of the median values, $\Sigma\frac{1}{2}$, have been computed from the dispersion $\pm(\Sigma\frac{1}{2}-\Sigma\frac{1}{6})$. The results are in Table III.

The data of Table II are graphically represented in Figure 1, curves I–IV, the points representing means of three successive values. The maximum frequency from these plots is entered in the last column of Table III. As was to be expected from the data in

TABLE II

FREQUENCIES OF LOG P IN DIFFERENT LONGITUDES

LOG P	GALACTIC LONGITUDE			ALL	MAG. CLOUD
	30°–150°	150°–270°	270°–30°		
0.00–0.20	....	....	....	....	4
0.20–0.25	....	1	....	1	2
0.25–0.30	1	....	....	1	6
0.30–0.35	1	....	....	1	5
0.35–0.40	....	....	....	....	....
0.40–0.45	....	....	1	1	5
0.45–0.50	1	1	....	2	5
0.50–0.55	....	1	1	2	4
0.55–0.60	3	3	....	6	7
0.60–0.65	4	3	1	8	9
0.65–0.70	3	5	1	9	12
0.70–0.75	5	5	1	11	6
0.75–0.80	5	2	3	10	4
0.80–0.85	1	4	6	11	5
0.85–0.90	1	2	4	7	1
0.90–0.95	1	1	3	5	2
0.95–1.00	....	1	4	5	4
1.00–1.05	4	2	4	10	1
1.05–1.10	2	1	....	3	2
1.10–1.15	....	1	4	5	3
1.15–1.20	2	2	....	4	3
1.20–1.25	4	1	3	8	4
1.25–1.30	1	2	1	4	....
1.30–1.35	....	....	2	2	....
1.35–1.40	....	1	....	1	....
1.40–1.45	....	3	....	3	1
1.45–1.50	....	....	....	....	2
1.50–1.55	....	1	1	2	2
1.55–1.60	2	1	....	3	....
1.60–1.65	1	1	....	2	1
1.65–1.70	1	1	....	2	....
1.70–1.75	....	1	....	1	....
1.75–1.80	....	....	1	1	....
1.80–1.85	....	....	1	1	1
1.85–1.90	....	....	1	1	....
Totals	43	47	43	133	....

TABLE III

MAXIMUM FREQUENCY AND MEAN ERROR

Gal. Long.	$\Sigma \frac{1}{6}$	$\Sigma \frac{1}{3}$	n	Mean Error	Max. Frequency
30°–150°	0.58	0.75	43	±0.03	0.70
150–270	.55	.78	47	.03	.70
270– 30	.74	.91	43	.03	.85
Mag. Cloud	0.38	0.62	103	±0.02	0.65

Table II, this is less than the median value, except in the case of the Small Magellanic Cloud. In the cloud a number of short-period stars produce a secondary maximum near $P = 2$, but in the

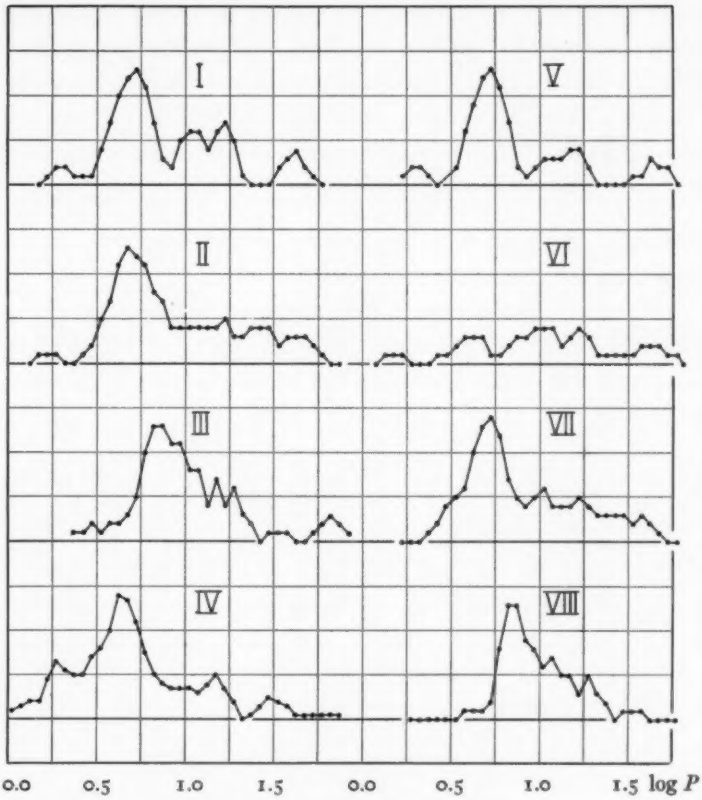

FIG. 1.—Frequency curves for galactic Cepheids

I. Gal. long. 30°–150°	V. Gal. long. 30°–110°
II. Gal. long. 150 –270	VI. Gal. long. 110 –180
III. Gal. long. 270 – 30	VII. Gal. long. 180 –300
IV. Small Magellanic Cloud	VIII. Gal. long. 300 – 30

For curve IV the scale of ordinates is one-half that of the other curves.

galaxy only a few Cepheids having these periods are known (such as SU and TU Cassiopeiae). In order to show still better the dependence of the frequency-curve on galactic longitude, the curves for somewhat narrower limits of longitude are given in Figure 1, curves V–VIII. The corresponding numerical data are in Table IV.

Sharp maxima are shown in Figure 1, V, VII, and VIII; for curve VI the number of stars is rather small as compared with the

TABLE IV

Curve	Gal. Long.	log P for Max. Freq.	Δm	Median Mag.	Corr. for Distance	No. of Stars
V.	30°–110°	0.70	0.0	8.8	−0.6	33
VI.	110–180	(.60)	+ .35	8.3	+ .25	21
VII.	180–300	.70	.0	8.2	.0	45
VIII.	300– 30	0.84	−0.49	8.2	−0.5	34

dispersion; but there is a suggestion of a maximum for a period shorter than for the other regions.

The period for maximum frequency is decidedly longer in the Sagittarius-Aquila region than in other parts of the galaxy, and shows a possible minimum near the opposite part of the sky (Fig. 1, curve VI).

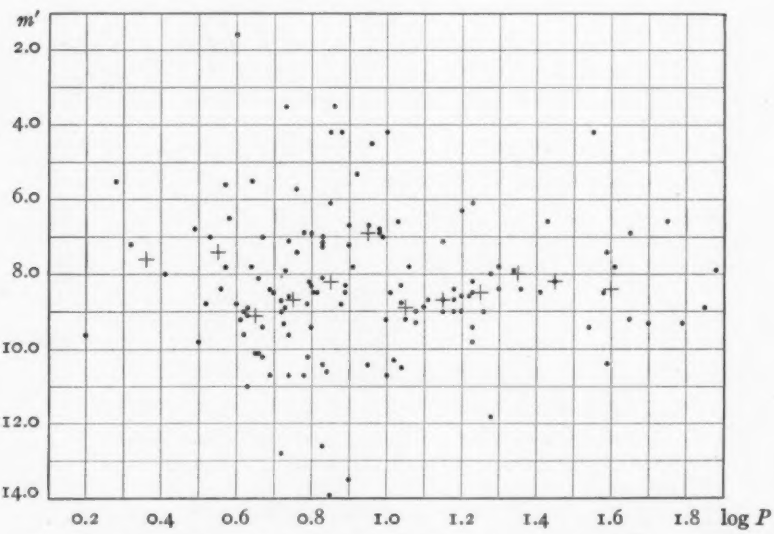

FIG. 2.—Apparent magnitudes of galactic Cepheids plotted against the logarithms of their periods. Crosses indicate median values of m' for equal intervals in log P.

Let us compare the apparent magnitudes of Cepheids with the logarithms of their periods. The magnitudes used are the means of the maximum and minimum values given in the "Katalog."

A constant color correction of -0.8 has been applied to photographic magnitudes. Since the periods, and presumably the absolute magnitudes, differ in different galactic longitudes, it is desirable to correct the apparent magnitudes for systematic differences in log P. The corrections used, which are given in the fourth column of Table IV, are based on the assumption that 0.1 in log P corresponds to -0.35 magnitude. The median apparent magnitudes given in the fifth column of the table are, moreover, to be reduced to a constant value, namely, 8.2. The total corrections finally applied to the apparent magnitudes are given in the sixth column.

TABLE V

MEDIAN MAGNITUDES AND PROPER MOTIONS

log P	Med. m'	Mean Error	No.	Med. $p.m.$	No.
<0.36	7.6	±0.6	6	0.022	3
0.5–0.6	7.4	.5	8	.026	5
0.6–0.7	9.1	.3	17	.028	4
0.7–0.8	8.7	.3	21	.019	6
0.8–0.9	8.2	.3	20	.015	10
0.9–1.0	6.9	.4	11	.013	6
1.0–1.1	8.9	.4	11	.031	3
1.1–1.2	8.7	.4	10	.015	1
1.2–1.3	8.5	0.4	11	.021	4
1.3–1.4	8.0	1.0	2	.031	2
>1.6	8.4	±0.3	16	0.025	9

The effect of these corrections, which is a reduction of the four galactic regions to the same median distance, is very small. They have been applied, however, in order to make it clear that systematic differences in the most frequent period and in the median apparent magnitude for the different galactic regions cannot be the origin of any possible correlation that may be found. The corrected magnitudes, m', plotted against log P, give the scatter diagram reproduced in Figure 2. The median value of m' for all periods together is 8.5 with a dispersion of ± 1.4.

Two things are at once evident: (1) the dispersion in m' is larger for stars having periods less than ten days ($\pm 1.8 \pm 0.2$, m.e.) than for those with periods greater than ten days ($\pm 0.95 \pm 0.14$, m.e.); (2) there is a lack of stars having log P between 0.9 and 1.0 and magnitudes between the limits 8.5 ± 1.4. The significance of this is more

clearly seen when we consider the median values of m' for successive intervals of log P, which are indicated by crosses. The median values and their mean errors, computed from the dispersion in m' for all stars together, are entered in Table V.

The difference in the median m' for the stars with log P equal to 0.95 and 1.05, respectively, is 2.0 ± 0.6 (m.e.). This difference in the median values for two groups of eleven stars each is remarkably large as compared with the dispersion of the individual magnitudes, viz., ± 1.4. The discrepant value 6.9 is, however, supported more or less by a gradual decrease of the three preceding values. The reality of this gradual decrease and the sudden increase of two magnitudes near log $P = 1.0$ can hardly be determined from the present material. For a combination in larger intervals of log P, we find the following median values of m':

log P	Median m'	No.
0.6–0.8	8.9	38
0.8–1.0	7.3	31
1.0–1.2	8.7	21

These show again that the median apparent magnitude for stars having log P in the interval 0.8 to 1.0 is systematically bright.

If the irregularity is not simply a result of selection in the data, it might be attributed (1) to peculiarities in the correlation of luminosity with period; (2) to irregularities in the density distribution of the stars. The latter would reveal itself through systematic differences in the median distances. Since some correlation between absolute and apparent magnitude is to be expected, the existence of a period-luminosity relation would imply a correlation between apparent magnitude and period. If $\Delta(\rho)$ and $\Phi(M)$ represent the density and luminosity functions, respectively, the number of stars having apparent magnitudes between m and $m+dm$, and absolute magnitudes between M and $M+dM$, will be

$$N(m)dm = \frac{4\pi}{3} dM \Phi(M) \Delta(\rho) \rho^2 d\rho \ , \qquad (1)$$

where

$$M = m - 5 \log \rho \ .$$

As an illustration, assume

$$\Delta(\rho) = e^{h + k(\log \rho) + l(\log \rho)^2} \ . \qquad (2)$$

Since

$$\rho^2 d\rho = a e^{3a(m-M)} dm \ , \qquad a = 1/Mod.,$$

(1) has the form

$$N(m)dm = \frac{4\pi}{3} \Phi(M) dM e^{\alpha + \beta m + \gamma m^2} dm \ .$$

Since $\Phi(M)$ is independent of m, the apparent magnitude for stars in the interval M to $M + dM$ has the mean value

$$\bar{m} = -\frac{\beta}{2\gamma} = -\frac{3a + 0.2k}{0.08l} + M \ , \qquad (3)$$

which does not depend on the luminosity function, and, aside from M, involves only the constants k and l of the density function.

For a linear period-luminosity relation

$$M = A + B \log P \ , \qquad (4)$$

we shall have a linear correlation between $\bar{m}$ and log P, provided the assumption (2) for the density law is justified. Something of the sort seems to apply in the case of stars having log $P < 1.0$. It is to be remarked, however, that $\bar{m}$ is extremely sensitive to small changes in the density law, for l is a small quantity, while the numerator of the first term in (3) is not small. The discontinuity in $\bar{m}$ near log $P = 1.0$ might therefore be ascribed to a difference in the density distribution of the stars having long and short periods. On the other hand, any deviation from the period-luminosity relation (4) would disturb the correlation of $\bar{m}$ with log P.

The second alternative, which would require greater distances for stars having log $P > 1.0$, makes it desirable to examine the known proper motions. This has already been done by R. E. Wilson,[1] who

[1] *Astronomical Journal*, **35**, 36, 1923.

found a continuous decrease in the parallactic motions of stars having periods up to forty days. He also found, however, that the τ-components *increase* for stars having $P>9$ days, and that the total proper motions for the stars of longer period are also systematically large, which is just the opposite of what is to be expected on the basis of the second alternative above.

Values of the median μ for the intervals of log P used here and the numbers of stars are given in the last two columns of Table V and illustrated in Figure 3. RY Boötis and W Virginis, having galactic latitudes $+62^\circ$ and $+57^\circ$, respectively, have been omitted, and also the stars in Wilson's list having the low weight 0.2. The

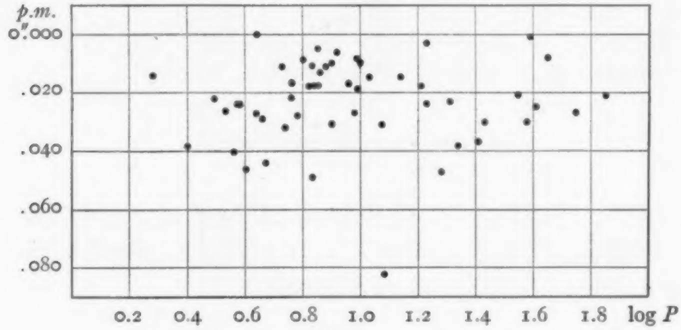

FIG. 3.—Proper motion plotted against log P for galactic Cepheids

omission of RY Boötis is justified on account of its spectrum, which does not show the Cepheid characteristics. The Cepheid character of W Virginis seems, however, to be well established, which disproves the rule that no Cepheids occur outside the galactic plane. The star has been omitted from the discussion because of its high velocity. On the other hand, stars having $P>40$ days are included in Table V. The median values for the fifty-three stars and the dispersion have been determined in the usual way: $\Sigma\frac{1}{2}$ = median value = $0''.021$; $\pm\frac{1}{2}(\Sigma\frac{5}{6}-\Sigma\frac{1}{6})$ = dispersion = $\pm\frac{1}{2}(0''.033-0''.010)=\pm0''.0115$. The median values show a decrease from log $P=0.6$ to 1.0, followed by a sudden increase near $P=10$ days, after which they are sensibly constant. In this respect their behavior is very similar to that of the apparent magnitudes m'. The mean differences between successive median values of μ are small with respect to their mean errors.

It is therefore advisable to combine the proper motions for larger intervals of log P than those used in Table V. Thus, for three groups we have

log P	Median μ	Corrected μ	No.
<0.8	0″025	0″024	18
0.8–1.0	.015	.014	16
>1.0	0.024	0.020	19

Since the motions of these stars as a class are small, their median values are appreciably affected by the systematic increase in the total proper motion arising from the effect of accidental error. Allowance, on the basis of the indicated probable errors, has been made for this in the "corrected" values.

The result is essentially the same as that found by Wilson. It should be remarked, however, that Wilson has shown the existence of a marked correlation between the size of the proper motion and its probable error. Since the indicated uncertainty for the third group is about twice that for the first two groups, this very suspicious circumstance may account for the unexpectedly large median for the long-period Cepheids.

The first attempt to determine the mean absolute magnitude for Cepheids was made by Hertzsprung[1] as early as 1906. He found from Auwers' proper motions for δ Cephei, ζ Geminorum, and η Aquilae the mean absolute magnitude -2.4. After the appearance of Boss's *Catalogue* he extended his material to thirteen stars, for which practically the same value was found (-2.3; sun's velocity, 20 km/sec.). Wilson has extended this material to forty-one stars, for which he finds a mean parallactic motion of 0″0142. Taking again the solar velocity of 20 km/sec., we find the mean absolute magnitude -1.0, which is surprisingly low. Meantime, the material for radial velocities has also increased. The thirteen Boss stars give a normal solar velocity of about 20 km/sec. in the direction of the general apex.[2] Recently Strömberg[3] derived a solar motion of 12.3 km/sec.

[1] *Zeitschrift für wissenschaftliche Photographie*, **5**, 94, 1907.

[2] The writer made a solution according to Bravais' method some years ago which has not been published.

[3] *Mt. Wilson Contr.*, No. 293; *Astrophysical Journal*, **61**, 363, 1925.

for Cepheids having periods longer than two days, which would raise the mean absolute magnitude from Wilson's parallactic motion to +0.1. The tendency of additional material, both for proper motion and radial velocity, to increase the absolute magnitude makes it worth while to treat the non-Boss stars separately. A graphical solution has been made for the parallactic motion of the non-Boss Cepheids having periods longer than one day, whereby $v/\sin\lambda$ was plotted against the weights, both quantities being taken from Wilson's paper. A solution for the sun's velocity in the galactic plane has also been made, based on the same material as used by Strömberg. The results for the assumed apex, galactic longitude 23°, latitude +22°, are as follows:

Non-Boss Stars	Boss Stars
$\Sigma\frac{1}{2}=q=+0\overset{''}{.}0095\pm0\overset{''}{.}0048$ (m.e.)	$\Sigma\frac{1}{2}=q=+0\overset{''}{.}013\pm0\overset{''}{.}0052$
$\Sigma\frac{1}{6}=-0\overset{''}{.}015$	$\Sigma\frac{1}{6}=-0\overset{''}{.}003$
$\Sigma\frac{5}{6}=+0\overset{''}{.}029$	$\Sigma\frac{5}{6}=+0\overset{''}{.}025$
Weight, 20.6	Weight, 8.3
$V^*=+6\pm2$ km/sec.	$V=+16\pm2$ km/sec.
Weight, 13.4 (28 stars)	Weight, 6.4 (13 stars)
$M=+1.3\pm1.6$	$M=-2.5\pm0.8$

* Solar velocity in galactic plane.

A factor 0.9 has been applied to the parallactic motions in order to reduce them to the galactic plane. Moreover, the present rather meager material for non-Boss stars shows a decrease of solar velocity with increasing period which counterbalances the decrease of parallactic motion as found by Wilson:

NON-BOSS STARS

Period	V	Weight	No. Stars	Paral. Motion	Weight	No. Stars
1^d–4^d	+23	1.9	5	$+0\overset{''}{.}020\pm.005$	3.0	6
4–10	9	4.7	11	$+\ .013\pm.005$	10.7	18
>10	+ 1	6.7	12	$-0.007\pm.007$	7.0	19

These results would indicate that the solar motion for the stars having periods longer than ten days is probably not in the direction

assumed. From a plot of the radial velocities it seems hopeless, however, to find any other direction.

A rough solution from the proper motions has been made, in which, for convenience, the stars have been assumed to be exactly in the galactic plane. Since the galactic latitudes are small, this will not materially affect the results. Components of the proper motion in the galactic plane, $\mu \sin \chi$, and at right angles to it, $\mu \cos \chi$, directed to the north galactic pole, have been computed with the aid of tables which Mr. Strömberg kindly put at my disposal. In order to include a possible rotation effect, a term W was introduced so that the equations of condition are of the form

$$x \cos \lambda + y \sin \lambda + W = \overline{\mu \sin x} \; .$$

The results indicate a rotation effect of $-0''.0075 \pm 0.0032$ (m.e.)[1] in the direction of Charlier's[2] rotation and about twice as large. A rotation of this order of magnitude is not a priori improbable, and might at least partly account for the existence of the proper motions. The parallactic motion in the galactic plane is $0''.0054 \pm .0038$ (m.e.), but does not justify any conclusion since the solar velocity in the same direction is inappreciable. From inspection, I found that the rotational effect is also indicated by stars having periods from about six to ten days, whereas for shorter periods there is no such indication. Repeating the foregoing calculation for all the non-Boss stars having periods longer than six days, which have been combined into seven groups, I find

$$\begin{aligned} x &= -0''.0068 \pm 0''.0026 \text{ (m.e.)} \\ y &= -0.0046 \pm 0''.0032 \\ W &= -0.0076 \pm 0''.0022 \end{aligned}$$

The equations for the components $\mu \cos \chi$ are

$$u \cos \lambda + v \sin \lambda + z = \overline{\mu \cos \chi} \; ,$$

[1] The mean error has been computed here on the assumption of a mean error of $0''.010$ in the p.m. having unit weight in Wilson's table.

[2] *Meddelanden från Lunds Astronomiska Observatorium*, Serie II, No. 9, 1913.

which give

$$u = -0''.0034 \pm 0''.0026 \text{ (m.e.)}$$
$$v = -0.0020 \pm 0.0032$$
$$z = -0.0088 \pm 0.0022$$

where z is the component of parallactic motion toward the north galactic pole, and u and v the rotations about two axes in the galactic plane. The latter are insignificant, whereas z is well above its mean error. When the apex is assumed to be in longitude 23°, the parallactic motion in the galactic plane and the rotational effect are

$$p = 0''.0000\text{; weight, } 9.9$$
$$W = -0''.0086\text{; weight, } 22.2$$

Assuming that the peculiar motions can be represented by $\mu \sin \chi + W$ and $\mu \cos \chi + z$, we find symmetrical frequency-curves for both components. For the combined frequency the observed dispersion is $\pm 0''.015$ and the corrected dispersion $\pm 0''.011$. The observed dispersion of the radial velocities for stars having the same periods (viz., > 6 days) is ± 16 km/sec. If we consider that part of the velocities rest on only a few observations, the mean error of a radial velocity will certainly not be overestimated by putting it ± 8 km/sec. In this case the corrected dispersion is ± 14 km/sec., from which the mean parallax is $4.74 \times 0''.011/14 = 0''.0037$, and the absolute magnitude 0.0.

Since, normally, the component of parallactic motion in the galactic plane is about three times that perpendicular to the plane, these results indicate either the existence of serious systematic errors in the proper motions or a systematic stream motion affecting the non-Boss stars having $P > 6$ days. The lack of uniformity in the distribution of the periods in longitude shown by the data in Tables II–IV and the curves in Figure 1 point toward a clustering of Cepheids, which would carry with it a tendency toward stream motion affecting individual groups. Until this question is settled, nothing positive can be affirmed as to the systematically low value found for the absolute magnitudes of these stars.

In connection with the first alternative on page 156, it is of inter-

est to recall that the spectroscopic absolute magnitudes of Cepheids,[1] which were derived by the criteria for stars of constant light and a mean spectrum corresponding to that of the Cepheids, do not show the range in luminosity to be expected on the basis of period-luminosity relation. A plot of the published data, together with magnitudes for six additional stars, kindly placed at my disposal by Mr. Adams and Mr. Joy, is shown in Figure 4, where the points represent individual stars. The plot shows an increase in brightness with log P from about 0.6 to 1.0. The median absolute magnitude of stars having periods longer than ten days is -1.5; for stars in the particular interval 0.8 to 1.0 it is -2.4;[2] in the interval 0.6 to 0.8,

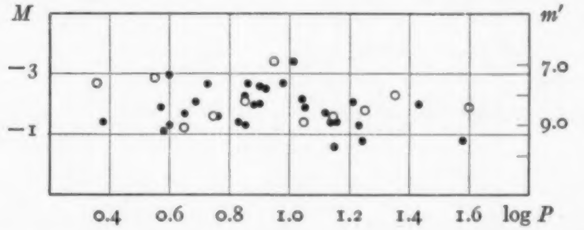

FIG. 4.—Spectroscopic absolute magnitudes of Cepheid variables. Open circles denote the median values of m' from Fig. 2.

-1.9; and for log P less than 0.6 it is -1.4. The circles in Figure 4 show the median values of m' taken from Figure 2; $m'=8.0$ has arbitrarily been made to coincide with -2.3 on the scale of the spectroscopic absolute magnitudes.

Although the material is scanty, it may be stated that a few special measurements on spectral lines with the microphotometer indicate a maximum in the intensity ratio of critical pairs of lines for a period of about ten days, followed by a drop in the ratio for stars having longer periods. The results are in Table VI. The tabulated quantity is the ratio of the intensity of an enhanced line to that of a neutral line. Settings were made at intervals of 0.01 mm on spectrograms having a dispersion of 36 A at $H\gamma$. The results have been plotted, and from these plots the intensities of the lines have been obtained.

[1] Adams, Joy, Strömberg, and Burwell, *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

[2] Including ζ Geminorum; period, 10.15 ± 1.05 days.

Although these results suggest an irregularity in the absolute magnitudes at a point corresponding to $P = 10$ days, it must be noted that the spectroscopic criteria used for the limited material published do not allow for differences in spectral type, which, in the case of the Cepheids, increases with P. This matter, which seems important, is under investigation by Mr. Adams and Mr. Joy.

The decrease of absolute magnitude for stars having periods from a little less than four days to about ten days is, curiously enough,

TABLE VI

RATIO OF INTENSITY OF ENHANCED TO NEUTRAL LINES FOR STARS OF DIFFERENT PERIODS

Star	Phase	Period	$\frac{4215}{4250}$	$\frac{4233}{4236}$	$\frac{4246}{4250}$	Mean for Each Plate	Mean for Two Plates
SZ Tauri	Max.	$3^d.15$	1.17	0.96	1.36	1.16	1.18
	Min.	....	1.28	.80	1.56	1.21	
RT Aurigae	Max.	3.75	1.13	.86	1.58	1.16	1.14
	Min.	....	1.05	0.92	1.37	1.11	
S Sagittae	Max.	8.38	1.24	1.02	1.43	1.23	1.20
	Min.	....	1.30	0.87	1.30	1.16	
ζ Geminorum	Max.	10.15	1.40	1.13	1.68	1.40	1.33
	Min.	....	1.17	1.08	1.53	1.26	
X Cygni	Max.	16.38	1.05	0.84	1.30	1.06	1.08
	Min.	....	1.08	0.92	1.28	1.09	
T Monocerotis	Max.	27.00	0.85	1.00	1.30	1.05	1.16
	Min.	....	1.38	1.46	1.00	1.28	

also shown by the early type binaries. Adams and Joy have pointed out that the Harvard spectral classification affords a basis for the determination of absolute magnitudes for stars of early type.[1] The absolute magnitudes for sixteen binaries having spectral types B0 to B3 on the Harvard scale, and for which periods are known, have been taken from the paper cited. Mr. Adams kindly permitted me to use the unpublished results for thirteen more stars, spectrograms of which have been taken mainly by Mr. Milton Humason. The material used thus consists of twenty-nine stars, of which twenty-four have periods longer than one day. The five remaining stars have periods between 3.16 and 0.26 days, and their ranges are exceedingly small. The absolute magnitudes are plotted in Figure 5

[1] *Mt. Wilson Contr.*, No. 262; *Astrophysical Journal*, **57**, 294, 1923.

against log P. This diagram shows a decrease in the mean M with increasing period from log $P = 0.6$ to somewhat more than 1.0. Still longer periods correspond to fainter magnitudes, and for periods greater than about twelve days the mean M seems no longer to vary with the period. The mean values of M are given in Table VII.

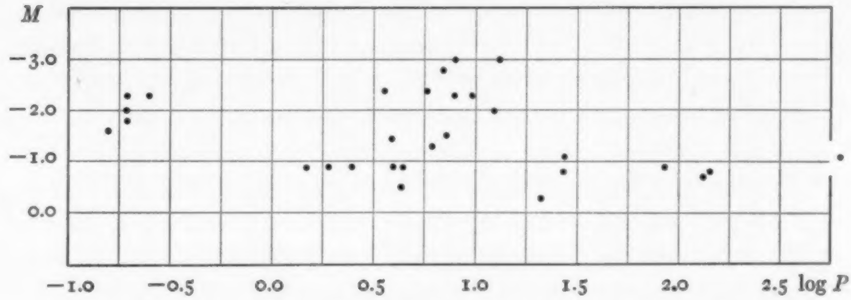

FIG. 5.—Spectroscopic absolute magnitude for spectroscopic binaries of types B0–B3 plotted against logarithm of period.

A few remarks may be added regarding the five stars having periods of about a fifth of a day. Several investigators have considered stars of this type to be pulsating stars rather than binaries, and, as Henroteau[1] has pointed out, there is an accumulation of

TABLE VII

PERIODS AND ABSOLUTE MAGNITUDES OF SPECTROSCOPIC BINARIES

log P	Mean M	No Stars
−0.7	−2.00	5
+0.29	0.90	3
.60	1.24	5
.80	2.00	4
1.00	2.52	5
1.53	0.78	4
2.37	−0.87	3

evidence requiring the consideration of these stars as Cepheids. If so, they can scarcely fit the period-luminosity law, because of their high luminosity. The proper motions are rather small, even for B stars, and the mean reduced magnitude $H = M + 5 \log \mu$ is equal to -6.74. Putting $H = -6.74 = -0.4 + 5 \log T - 8.38$, where T is the

[1] *Publications of the Dominion Observatory*, **9**, 1, 1925.

linear tangential velocity and -0.4 the adopted absolute magnitude, we find $T = 2.5$ km/sec., which is an improbably low value, although the mean radial velocity, without regard to sign, is not very large, viz., 15 km/sec.

As they stand, the data from various sources—median apparent magnitudes, proper motions, radial velocities, and spectroscopic absolute magnitudes—appear to indicate a discontinuity or irregularity in the various characteristics corresponding to a period of about ten days. Were the data themselves above suspicion, we should be tempted to say that the galactic Cepheids are composed of two classes. But in no case is the evidence convincing. It is not certain that the apparent magnitudes form a representative collection; the proper motions may be affected by systematic error, and those of the questionable stars of long period are relatively uncertain, perhaps to a much greater degree than indicated by the calculated probable errors; the radial velocities are too few in number to give a trustworthy value of the all-important solar motion; and, finally, it is probable that the usual spectral criteria for luminosity do not apply to Cepheids unless differences in spectral type are taken into consideration.

Some of these difficulties may be associated with the more positive results of the discussion, namely, the lack of Cepheids having short periods in the Sagittarius-Aquila region and the evidence of peculiarities in the radial velocities, both of which indicate a lack of homogeneity that is perhaps to be explained by a clustering of stars into groups affected by stream motion. Until these questions have been cleared up it is evidently premature to assert dogmatically that the present assumptions concerning the absolute magnitudes of the Cepheids are either right or wrong. Perhaps the most important result, however, is the indication brought out by the discussion of the urgent necessity for additional data on both proper motions and radial velocities.

I wish to express my indebtedness to Mr. Seares for the development of the relation between mean apparent magnitude, density, and absolute magnitude given on page 157.

MOUNT WILSON OBSERVATORY
June 1926

NEW TERMS IN THE SPECTRUM OF CALCIUM

By R. J. LANG

ABSTRACT

The vacuum-spark spectrum of Ca shorter than 2000 A has been photographed with a 6-foot grating *in vacuo* having 30,000 lines per inch and giving an average dispersion of 4.5 A per mm.

Some of the *known lines* of *Ca*+ have been *remeasured*, especially those of the *first fundamental series*, and compared with the values published by Saunders and Russell.

Two new terms are given which combine with many of the known terms, especially with the dashed terms.

The spectra of the normal and singly-ionized calcium atoms have received a great deal of attention, and the main features of these spectra are now rather thoroughly known. The work of Saunders[1] on the normal atom and Saunders and Russell[2] on the ionized atom are especially noteworthy. The vacuum-spark spectrum of wave-length shorter than 2000 A was published by Millikan and Bowen[3] in 1924, and in the same year J. A. Anderson[4] mapped the vacuum-spark spectrum from 2000 to 6000 A. Later still, Russell and Saunders[5] found new combinations between many of the ordinary known terms and certain new terms which they denoted by "dashed" symbols. These combinations resulted in groups of lines now commonly known as "multiplets."

The present paper gives the results of a re-examination of the *Ca* spectrum in the Schumann region by means of a vacuum spectrograph, already fully described elsewhere.[6] The concave grating has a radius of 6 feet and 30,000 lines per inch giving an average dispersion of approximately 4.5 A per mm. This grating was supplied by Professor R. W. Wood, of Johns Hopkins University. It gave very excellent spectra longer than about 1000 A, but shorter than that value the intensity of the spectra was poor, and it failed entirely at about 500 A. The calcium used was purchased from Messrs. Eimer and Amend and showed as impurities chiefly silicon and a trace of aluminum. The question of impurities in the source

[1] *Astrophysical Journal*, **52**, 265, 1920.

[2] *Ibid.*, **62**, 1, 1925.

[3] *Physical Review* (2), **23**, 1, 1924.

[4] *Astrophysical Journal*, **59**, 76, 1924.

[5] *Ibid.*, **61**, 38, 1925.

[6] *Journal of the Optical Society of America*, **12**, 523, 1926.

is, however, one of the chief difficulties connected with the high-potential spark as a small trace of a foreign substance may result in the lines of its spectrum, especially for some stages of ionization, appearing very strongly.

The spectra were taken on Schumann plates of excellent quality supplied by Adam Hilger, Ltd. These were measured on a comparator reading to 0.001 mm, those lines of carbon, hydrogen, etc., which were standardized by S. Smith and the author[1] being used as standards. One exception consists in the fact that the *Al* line 1862.69 was used in some cases.

The source of radiation was the high-potential spark *in vacuo* produced by a large X-ray coil, the primary of which was joined through suitable resistance to the 110-volt A.C. 60-cycle mains. The secondary of the coil and two spark-gaps were connected in series, one gap being in air and the other *in vacuo*. The gap in air was about 1 cm wide between brass spheres of about 2.5 cm in diameter, and the gap *in vacuo* was about 1 mm or less. Across the secondary of the coil five large Leyden jars of about 2-gallon size were connected in parallel. The energy in the secondary circuit could not be measured directly with the apparatus available, but the potential was estimated by means of a sphere-gap at between 30,000 and 50,000 volts. The maximum primary current in the coil was about 30 amperes.

New measures of Ca^+.—Such of the known lines of *Ca* as fall within the region investigated and which appeared on these plates were remeasured. These belonged entirely to Ca^+ and will be discussed here in connection with the values given by Saunders and Russell.[2] As the values quoted by them shorter than about 1850 A were presumably measured on a 1-m grating having about 14,000 lines per inch, the dispersion used in this work corresponds roughly to the fourth order.

No members of the first sharp or first diffuse series could be found, but those of the first fundamental series came out rather well. In Table I the values from this work are listed, together with those of Saunders and Russell.

The new measures are all slightly shorter than the old, and the wave-number separations are much more nearly constant at a value

[1] *Physical Review* (1), **28**, 36, 1926. [2] *Astrophysical Journal*, **62**, 1, 1925.

of about 60. This is especially true for the second and fourth pairs listed. This separation was measured many times, especially for the fourth pair, which is very faint on these plates. Hence, while the measurements of wave-lengths may possibly be astray slightly, there is very little doubt concerning this separation being close to 60 units.

Some doubt exists regarding the second pair of the first principal series of Ca^+. Saunders and Russell give the values 1649.96(2) 60607.5; 1652.02(1) 60532.0, or a separation of 75.5. Two lines appeared persistently on these plates at 1649.94(3), 60608, and 1647.49 (2) 60698, having a separation of 90. These lines are taken to be

TABLE I

Lang				Saunders and Russell		
λ I.A. Vac.	I	ν	Δν	λ Vac.	ν	Δν
1839.96.....	10	54349	62	1840.21	54341.6	63
1837.89.....	9	54411		1838.08	54404.6	
1554.70.....	3	64321	60	1555.1	64304	66
1553.25.....	2	64381		1553.5	64370	
1433.72.....	1	69749	58	1434.2	69720	58
1432.52.....	0	69807		1433.1	69778	
1369.50.....	00	73019	63	1370.6	72960	80
1368.33.....	00	73082		1369.1	73040	

combinations between the 1p′-levels and a new level as shown later. The reason for this is that the intensities are definitely in the wrong order for these to belong to the P series, as well as the fact that, unless the wave-lengths quoted by Saunders and Russell are very badly astray, these lines are of definitely shorter wave-length. This pair of lines was measured in two different settings of the grating: first, with 1400 A at the center of the plate with the hydrogen line 1215.68 and the carbon line 1561.46 as standards; and second, with 1800 A at the center of the plate and 1561.46 and 1862.69 as standards. Several plates were taken in each setting, and the greatest deviation from the mean among these measures was 0.04 A. There was no sign of a line appearing at 1652. No trace could be found of the third members at 1342 A. On the other hand, one would expect the members of the P series of Ca^+ to appear, and the first pair at 3933 A came out strongly on the plates taken by Anderson.

New terms.—In the Schumann spectrum of *Ca* as revealed by this high dispersion several faint pairs and groups of lines appeared which do not belong to any combinations between known terms. It was noted that the separations involved in several of these correspond to the separations of known multiple terms, especially of those de-

TABLE II

Combination	Theoretical Values		Observed Values		
	λ	ν	λ I.A. Vac.	I	ν
$V_1-2\delta_1$	3079.67	32,471	3079.10	2*h*	32,477
$V_2-2\delta_1$	3073.99	32,531	3073.18	1	32,540
$V_2-2\delta_2$	3072.19	32,550	3071.39	1	32,558
V_2-1S	4517.73	22,135	4516.60	8	22,141
$1\sigma-V_2$	4115.39	24,299	4116.04	3	24,295
V_1-W	1872.44	53,406	1872.27	12	53,411
V_2-W	1870.34	53,466	1870.17	12	53,471
V_2-x	1725.00	57,971	1724.91	1	57,974
V_2-1p_3'	1651.44	60,553	1651.19	00	60,562
V_2-1p_2'	1650.16	60,600	1649.94	3	60,608
V_2-1p_1'	1647.79	60,687	1647.49	2	60,698
V_2-Z	1591.70	62,826	1593.81	1	62,743
V_2-X	1590.96	62,855	1592.94	6	62,777
V_2-3f_{123}	1555.06	64,306	1555.58	10	64,285
V_1-2p_1'	1415.61	70,641	1415.77	00	70,633
V_2-2p_2'	1414.92	70,675	1414.90	2	70,676
V_2-2p_3'	1414.40	70,701	1414.40	2	70,701
V_2-2d_1'	1353.10	73,904	1352.95	00	73,913
V_1-3p_3'	1308.22	76,440	1309.28	0	76,378
V_2-3p_1'	1309.59	76,360	1310.72	0	76,294

noted by dashed symbols. A study of these lines and groups of lines led to the discovery of two new terms which appear to combine with many known terms. In Table II these new levels are denoted by the symbols $V_1=71{,}380$, $V_2=71{,}440$. These symbols are chosen so as not to suggest anything regarding the type to which these terms belong, since the data at present available do not seem sufficient to determine this.

Table II gives the combinations which are apparently involved,

together with the theoretical value of the wave-length and wave-number which the lines should have on the assumption of the existence of the two levels given above. With these are listed the corresponding values of lines which have been found in the spectrum. The values above 2000 A are taken from Anderson's paper. In one or two cases the agreement is hardly within the experimental error involved. Thus the combinations between the X- and Z-levels and the new levels are rather doubtful.

TABLE III

UNCLASSIFIED GROUPS

λ I.A. Vac.	I	ν	Δν	λ I.A. Vac.	I	ν	Δν
1658.66	0	60,290	....	1378.98	3	72,517	60
1657.90	0	60,317	....	1377.86	2	72,576	
1656.89	1	60,354	....				
1656.19	0	60,379	....	1361.23	1	73,464	14
1655.51	0	60,404	....	1360.95	1	73,478	
1654.59	0	60,438	....	1360.55	2	73,500	26
				1360.04	1	73,526	
1417.28	1	70,558	12				
1417.02	1	70,570		1327.16	0	75,349	42
				1326.42	1	75,391	
1385.51	1	72,176	68				
1384.19	0	72,244		1323.97	2	75,530	45
1383.21	0	72,296	52	1323.25	2	75,575	
				1321.65	0	75,663	35
				1321.04	0	75,698	

In Table III are listed some other pairs and groups of lines, the separations of which in certain cases look familiar, but of which no sure combination could be found. The group at 1360 A is especially clear, and the measures, as given, are rather accurate on that account. It is seen that the separations of the 2p′-levels are clearly involved, but no corresponding combinations for the 1p′- or 3p′-levels could be found. The group at 1656 was not so accurately measured on account of its faint and apparently diffuse nature. The combination of the 1d′-levels with V_2 gives the position of some of the lines in this group approximately, but no definite conclusions could be reached. The line 1657.90 seems to be really double.

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ON THE ORBITS OF FOUR SPECTROSCOPIC BINARIES[1]

By R. F. SANFORD

ABSTRACT

Orbits of spectroscopic binaries.—Four systems, η Orionis A, ϵ Hydrae C, Boss 4247, and B.D.$+57°2309$ are studied by means of slit spectrograms of one-prism dispersion made for the most part with the large reflectors at the Mount Wilson Observatory.

η Orionis A.—The velocity of the center of mass varies in a *period* of *9.5 years* with a semi-amplitude of about 18 km/sec. Beal's earlier estimate of between 9 and 10 years for the period of variation in γ is thus confirmed. If explainable as a triple system, the velocity of the system is about $+20$ km/sec. The three *masses* would be comparable and of the order of 10 ☉ for each as a minimum. Radial velocities from the *D lines* of sodium are *constant,* thus agreeing with Slipher's observations for the H and K lines and exhibiting what has already been found for several other spectroscopic binaries of early B spectrum. Velocities for the triple system, for the D lines, and for the Orion nebula are in fairly good agreement, which may be significant since η Orionis is near the inner edge of the large ring of nebulosity surrounding the Orion nebula.

ϵ Hydrae C.—The orbit found for this member of the well-known visual triple has an *eccentricity* among the *largest* associated with spectroscopic binaries and comparable with that of the orbit of AB of the same system. The period is 9.9 days; the velocity of the system is about 6 km/sec. less than that found for A and B, which may be a consequence of the motion of C about the pair AB.

Boss 4247.—Since the lines of both spectra appear, it has been possible to determine the orbital elements of *both components* of the spectroscopic binary. The period is 2.3 days; the orbits are *circular,* and the two semi-amplitudes of velocity variation are 97.4 and 108.7 km/sec. These data give 1.11 and 0.99 ☉ as the values of $m \sin^3 i$ and $m_1 \sin^3 i$, respectively.

B.D.$+57°2309$.—This star is *imbedded in nebulosity.* Its spectral class is B3. The radial velocity varies in *two periods:* one 5.4 days, the other 225 days. Both orbits are elliptical, with small eccentricities; the semi-amplitudes of velocity variation are 40 and 22 km/sec., respectively. If interpreted as a *triple system,* the velocity of the center of mass would be -17.2 km/sec. The H and K lines are strong and sharp, and are provisionally accepted as showing a *constant* velocity of -19.4 km/sec., which, there is reason to believe, is the velocity of the surrounding nebulosity. Since the difference between this mean and the velocity for the assumed triple system can easily be accounted for by their respective probable errors, equality may be assumed to exist, whence it follows that the *triple system is at rest within the nebulosity.* Some evidence indicates that the D lines also do not oscillate with the lines which furnish the data for the two orbits found.

This paper derives orbits from the variable radial velocities of the four stellar systems listed in Table I. In three cases the data consist wholly of spectrograms made with the one-prism spectrographs attached to the 60- and 100-inch reflectors in Cassegrain form, while in the fourth they also include earlier measures, antedating the Mount Wilson series. The second, third, and fourth

[1] *Contributions from the Mount Wilson Observatory,* No. 317.

binaries were discovered at this Observatory.[1] The introduction to a previous paper[2] gives general details concerning instruments, observations, and methods of reduction.

η ORIONIS A AS A TRIPLE SYSTEM

η Orionis (βG.C. 2712) is a stellar system whose complexities have been resolved one by one by various observational methods as astronomical equipment has improved. Sir William Herschel in the latter part of the eighteenth century noted a tenth-magnitude companion, to which Burnham ascribes a proper motion identical with that of η Orionis. In the middle of the last century Dawes found the bright star to be a close double, the magnitudes of the

TABLE I

Star	H.D. No.	Mag.	α 1900	δ 1900	Spectrum
η Orionis A..............	35,411	3.7	$5^h19^m.4$	$-\ 2°29'$	B1
ϵ Hydrae C..............	....	7.5	8 41.5	+ 6 47	F5
Boss 4247..............	150,682	5.9	16 37.5	+27 7	F2,F2
B.D.+57°2309..........	203,025	6.4	21 14.6	+58 10	B3

components A and B being 3.7 and 4.9, respectively. Orbital motion has not been detected with certainty, the position angle and distance remaining about 80° and 1″, respectively.

Frost and Adams[3] at the Yerkes Observatory found the radial velocity to be variable. About a year later, Adams[4] published an orbit for the spectroscopic binary which is so nearly circular that it may be described by the period 7.9896 days; the semi-amplitude of velocity variation, 144.75 km/sec.; and the epoch of maximum negative velocity, J.D. 2415723.849. These give a very good representation of the Yerkes observations.

The pair is so close that integrated light from both stars passes into the slit of the spectrograph, but the difference of magnitude is such that the resulting spectrum is essentially that of star A. Its spectral class is B1.

[1] *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

[2] *Mt. Wilson Contr.*, No. 201; *Astrophysical Journal*, **53**, 201, 1921.

[3] *Publications of the American Astronomical Society*, **1**, 179, 1902.

[4] *Astrophysical Journal*, **17**, 68, 1903.

In 1909 and again in 1915, a series of spectrograms of η Orionis A was made at the Allegheny Observatory. A. F. Beal[1] measured these and found good agreement with Adams' elements, provided it was assumed that γ, the velocity of the center of mass, varied in a sine curve with a period between nine and ten years. Many of the Allegheny plates showed the spectra of two components, from which Baker[2] obtained 10.6 and 11.2 $\odot$ as the two mass functions pertaining to the short period.

A half-dozen spectrograms made by Slipher[3] at about the same epochs as the Allegheny series gave a constant velocity for the sharp H and K lines of calcium.

Next in order came observations by Stebbins[4] with a photoelectric cell, which showed η Orionis to be an eclipsing binary with a principal minimum of at least 0.15 mag. below normal light and a secondary minimum of 0.02 or 0.03 mag. Stebbins states that "eclipses last probably less than half a day and seem to occur slightly before the predicted times." The period is so nearly an exact number of days that the complete photometric observation of the star becomes a slow process; but such results will be extremely valuable for the determination of the inclination of the orbit of the spectroscopic binary, and hence of the true semi-major axis and the masses.

A set of thirteen spectrograms was made at the Vienna Observatory in 1920, which Hnatek[5] combined with the early Yerkes plates for a redetermination of the elements. The results agreed well with those by Adams, the period, however, being increased 0.0034 day.

Still another series of radial velocity determinations has been made by the writer at Mount Wilson about the mean epoch 1925.0, the details for which are given in Table II. Phase is reckoned from J.D. 2415723.849, the epoch found by Adams for maximum negative velocity.

Since both the Allegheny and Vienna plates indicate changes in the elements, it seemed best to discuss all the radial velocities to-

[1] *Publications of the American Astronomical Society*, **3**, 117, 1915.

[2] *Lick Observatory Bulletins*, **11**, 170, 1925.

[3] *Ibid.*

[4] *Publications of the American Astronomical Society*, **3**, 272, 1916.

[5] *Astronomische Nachrichten*, **217**, 53, 1922.

gether. The values for the Yerkes, Vienna, and Mount Wilson Observatories were at hand. Dr. Curtis was kind enough to send the twenty-seven determinations comprising the Allegheny series of 1909 and the seventeen for 1915, with permission to make use of them. Dr. V. M. Slipher was so good as to send me his six plates for measurement. These have been used only as a confirmation of the much longer Allegheny series, made at the same epochs.

Each series of velocities was projected back upon Adams' velocity-curve with his period, from which it became apparent that

TABLE II

MOUNT WILSON OBSERVATIONS OF η ORIONIS A

Plate	Date	G.M.T.	Phase	Vel.	Corr. for Variable γ	Corrected Velocity
				km/sec.	km/sec.	km/sec.
γ 12196	1923 Oct. 21	$0^h 28^m$	$0^d.982$	−89	+22.5	−66.5
γ 13034	1924 Oct. 17	0 17	3.428	+131	+28.5	+159.5
γ 13048	Oct. 18	23 28	5.397	+75	....	+103.5
γ 13060	Oct. 19	23 32	6.397	−19	....	+9.5
γ 13088	Nov. 7	23 47	1.439	−60	....	−31.5
γ 13118	Dec. 2	19 26	2.291	+29	....	+57.5
γ 13124	Dec. 3	21 45	3.388	+119	....	+147.5
γ 13128	Dec. 8	19 2	0.285	−122	....	−93.5
γ 13140	Dec. 15	17 42	7.130	−93	....	−64.5
γ 13171	1925 Jan. 6	18 25	5.292	+84	....	+112.5
γ 13173	Jan. 6	19 8	5.332	+82	....	+110.5
C 3121	Jan. 9	18 34	0.309	−134	....	−105.5
γ 13195	Jan. 10	18 48	1.318	−77	....	−48.5
γ 13769	Sept. 27	23 59	5.879	+16	+29.5	+45.5
C 3587	Dec. 2	20 7	7.802	−137	....	−107.5
γ 3591	Dec. 3	20 36	0.834	−118	....	−88.5
γ 14052	1926 Feb. 24	18 21	3.804	+133	....	+162.5

only a slight shortening of his period would make all the series agree (except the Vienna observations), with γ alone variable. An increase of 0.0034 day is the minimum change in Adams' period which will reconcile the Yerkes and Vienna observations; and since this change is inadmissible for all the other series, the Vienna observations have been ignored, except in determining the value of γ. The new period, 7.98922 days, which harmonizes the remaining series, is 0.00038 day shorter than Adams' original period. This is not to be interpreted as a real change in the period, but as the improvement contributed by the long interval now available. In qualitative

agreement with this modification is the fact noted by Stebbins that eclipses occur before the epochs predicted with the old period.

With each series plotted against Adams' curve, the value of γ for each set of plates was easily found by the mean departure from Adams' γ. Two series of plates, which have been made since Beal found nine to ten years as the probable period for the change in γ, confirm his estimate, for all the values fall on a smooth curve with $P=9.5$ years, as shown in the lower part of Figure 1. Table III lists the separate values of γ.

The values do not define a velocity-curve with sufficient precision to distinguish between a circular and an elliptic orbit, although, as plotted, some eccentricity is indicated. This uncertainty affects to

TABLE III

VALUES OF γ FOR η ORIONIS A

No.	Epoch	Vel.	Series
		km/sec.	
1...........	1902.4	+35.5	Yerkes
2...........	1909.4	+24.8	Allegheny
3...........	1915.0	+ 9.4	Allegheny
4...........	1920.2	+37.0	Vienna
5...........	1925.0	+ 7.2	Mt. Wilson

some extent the mean γ, which was obtained in the usual way by finding the line which divides the γ-curve into equal areas. This line corresponds to +20 km/sec., which is probably a fair approximation to the velocity of the system, if triple. The semi-amplitude of velocity variation is about 17.5 km/sec.

The ordinates of the curve for γ give the corrections required to reduce each series of observations to Adams' curve. The upper part of Figure 1 shows this curve, the two series of Allegheny observations thus corrected and combined into normal places, and, finally, my individual values similarly corrected for variation in γ. Phase is reckoned from greatest negative velocity. Although minor improvements might still be made, the agreement is satisfactory. The Vienna observations, if plotted, would be well represented by Adams' curve displaced 3.1 days to the right, no adequate explanation for which has been found.

To predict the radial velocity of η Orionis A, it is therefore only necessary to use Adams' elements with the new period, 7.98922 days, and modify the velocity thus found by the difference between Adams' value of γ and that found from the curve in the lower part of Figure 1. This seems the safest procedure, since to give elements for the variation in γ would perhaps suggest a degree of accuracy certainly not at present attainable.

If we assume that the spectroscopic system involves three bodies and that the inclinations of both short- and long-period orbits are

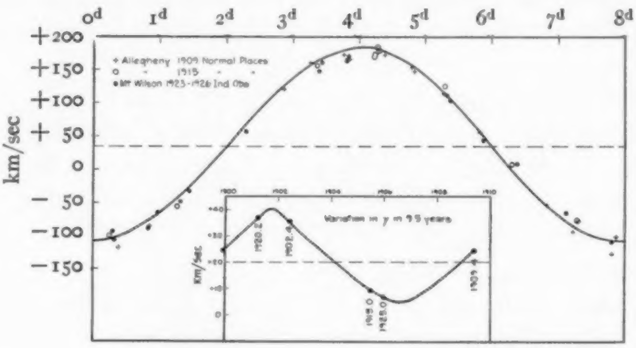

FIG. 1.—Radial velocity curves for η Orionis A

nearly 90°, then it follows that the mass function for the long-period orbit is

$$\frac{M_1^3 \sin^3 i}{(M+M_1)^2} = 1.9 \odot ,$$

where M is the combined mass of the two stars whose period is eight days, and M_1 the mass of the third star. But if $i=90°$, $M=(10.6+11.2)\odot$, whence M_1 is of the order of $13 \odot$, so that we appear to be dealing with three comparable masses.

These three components comprise star A of the visual system, magnitude 3.7, which is 1 mag. brighter than the value usually assigned to star B. It is of interest that spectra of two of the stars in A are shown on many spectrograms without serious interference from the spectrum of B. Of course the latter may be fainter than estimated, at least photographically. At any rate, the differences in magnitude must be most favorably distributed to insure the selec-

tion actually found upon spectrograms resulting from the combined light of all four stars. It is hoped that with first-class seeing conditions the spectrum of B may be obtained free from that of A.

Spectroscopic binaries of type B3, or earlier, are quite numerous in which the H and K lines of calcium are sharp and give velocities which do not seem to vary. In several such cases, Miss Heger[1] has found a similar behavior for the D lines of sodium. Since Slipher finds that the H and K lines in η Orionis A give a constant velocity, I have obtained two plates covering the region of the D lines. The dispersion of the plane-grating spectrograph used for this purpose is 65 A per millimeter. A neon comparison spectrum was used. From

TABLE IV

η ORIONIS A—D-LINE VELOCITIES

Plate No.	Date	Computed Vel.	Meas. Vel. D_3	Meas. Vel. D_1 and D_2
		km/sec.	km/sec.	km/sec.
G113.......	1926 Jan. 27	−137	−134	+26.8
127.......	Mar. 27	+ 82	+ 51	+ 6.6
Mean...	....	....	....	+16.7

two measures of each plate, I have derived one velocity from D_3 of helium and another from the D lines of sodium, and have also computed the velocity from the elements already found. The results are in Table IV. The agreement between the computed velocities and those from D_3 is quite as good as one could expect from a single rather weak line and such dispersion. Although the phases are such as to give a large difference in the orbital velocities, the results for the D lines are not greatly different, and their mean, +17 km/sec., is not far from +20 km/sec., which has been found for the velocity of the triple system. It is of interest that η Orionis is located on the inner edge of the great ring of nebulosity surrounding the Orion nebula, and that the velocities from the D lines as well as for the triple system are nearly the same as the mean of many determinations for the Orion nebula itself (+17 km/sec.) as published by Campbell and Moore.[2]

[1] *Lick Observatory Bulletins*, **10**, 59, 1919; *ibid.*, p. 141, 1921.

[2] *Lick Observatory Publications*, **11**, 107, 1918.

THE ORBIT OF THE SPECTROSCOPIC BINARY ϵ HYDRAE C

The multiple system, ϵ Hydrae (βG.C. 4771), is of more than ordinary interest, for it includes a visual pair whose data were complete enough in 1912 to enable Aitken[1] to derive both its visual and spectrographic orbits, and thus obtain the linear dimensions of the orbit and the parallax. In addition, there is a third star, C, at a distance of $3''$, of about 7 mag., which has described an arc of some $40°$ about AB during the one hundred years since its discovery. The spectrum obtained from the close visual pair (separation $0\overset{''}{.}23$) is F8, and is that of the brighter component A. The spectral class of star C is F5.

To obtain the spectrum of C free from that of AB, it is necessary to keep the image very nearly at the same point on the slit, thereby producing a very narrow spectrum. Furthermore, the seeing must be good; otherwise the enlarged images merge and the spectrum becomes a composite of A, B, and C. Effort has always been made to meet these requirements, but some of the measures may be affected by blending with the brighter star, either because the rate of the driving clock made close guiding very difficult or because the seeing was variable throughout the exposure.

The first two spectrograms early in 1917 gave velocities which differed from each other and from the value of the velocity of the center of mass for AB so much and in such a direction that the binary character of C seemed certain. The seventh and eighth plates, taken in December, 1922, definitely settled the matter, after which the star was placed on a selected list of spectroscopic binaries for regular observation.

In all, forty-four spectrograms were obtained, covering satisfactorily the entire velocity-curve. Less than one-sixth of the period is required for the drop from maximum to minimum velocity; this and the other conditions already referred to delayed considerably the completion of the observations covering this branch of the velocity-curve.

The final adjustment of the observations gave $P = 9\overset{d}{.}9047$, which is judged to be correct within 0.001 day, since any greater change

[1] *Publications of the Astronomical Society of the Pacific*, **24**, 216, 1921.

TABLE V

OBSERVATIONS OF ϵ HYDRAE C

Plate	Date	G.M.T.	Phase	Velocity	O−C	Wt.	Notes
				km/sec.	km/sec.		
γ 5361....	1916 Dec. 12	22^h31^m	$5^d.962$	+38.7	− 4	½	(1)
γ 5419....	1917 Jan. 4	21 54	9.128	+56.7	+ 3	1	
γ 5573....	Feb. 9	20 35	5.454	+43.7	+ 4	1	
γ 7771....	1919 Jan. 15	20 08	7.201	+44.8	− 5	1	
γ 7946....	Mar. 11	20 11	2.774	+20.7	− 2	1	
γ 8038....	Apr. 11	17 15	3.938	+30.2	− 1	1	
γ 11456....	1922 Dec. 5	00 25	0.103	− 5.8	− 1	½	(2)
γ 11463....	Dec. 5	22 51	1.038	−10.0	− 9	1	
γ 11498....	1923 Jan. 2	22 01	9.193	+43.9	− 9	¼	(2), (3)
γ 11543....	Jan. 8	22 01	5.290	+34.0	− 6	½	(3), (4)
γ 11576....	Jan. 27	20 18	4.407	+42.2	+ 8	1	
C 2100....	Jan. 31	23 07	8.520	+51.8	− 3	¼	(3), (4)
γ 11616....	Feb. 7	18 35	5.431	+40.6	0	¼	(3), (4)
γ 11625....	Feb. 24	18 39	2.626	+25.8	+ 5	1	
γ 11645....	Mar. 1	20 48	7.711	+46.3	− 6	½	
γ 11655....	Mar. 5	17 18	1.663	+ 7.4	− 2	1	
γ 11748....	May 2	16 30	0.203	−16.3	− 6	1	
C 2236....	May 4	15 18	2.153	+16.4	+ 1	1	
γ 11759....	May 5	15 45	3.172	+25.4	0	1	
γ 11766....	May 6	15 20	4.154	+26.6	− 6	1	
γ 11772....	May 7	15 48	5.173	+36.5	− 2	1	
C 2280....	May 31	15 42	9.361	+54.9	+ 6	1	
C 2546....	Nov. 22	0 55	5.459	+43.1	+ 2	1	
γ 12304....	Nov. 25	21 35	9.416	+52.6	+ 6	1	
C 2599....	Dec. 28	21 28	2.696	+18.6	− 3	½	(1), (3)
γ 12367....	1924 Jan. 12	21 52	7.810	+51.7	− 1	1	
C 2606....	Jan. 12	23 54	7.884	+49.0	− 4	½	(1)
γ 12407....	Jan. 17	22 40	2.937	+27.6	+ 4	1	
C 2643....	Jan. 20	19 46	5.817	+50.4	+ 8	1	
γ 12449....	Jan. 26	18 57	1.878	+ 8.7	− 3	1	
γ 12565....	Apr. 16	15 16	3.486	+22.1	− 6	1	
γ 12578....	Apr. 18	15 21	5.491	+55.7	+15	½	(2), (3)
γ 13174....	1925 Jan. 6	19 40	1.243	+ 8.3	+ 5	1	
γ 13382....	Apr. 14	15 14	0.012	− 6.8	−10	1	
γ 13417....	May 6	15 43	2.222	+16.8	0	1	
γ 13430....	May 14	15 53	0.324	− 0.9	+12	1	
γ 13996....	1926 Jan. 25	21 54	8.959	+52.1	− 3	1	
γ 14002....	Jan. 26	17 20	9.768	+20.9	+ 2	1	
γ 14006....	Jan. 26	21 21	0.031	− 4.8	− 6	1	
γ 14014....	Jan. 27	21 22	1.031	− 2.9	− 2	1	
C 3677....	Feb. 5	19 08	0.033	+ 8.2	+ 7	1	
γ 14126....	Mar. 27	17 26	0.438	−14.3	− 2	1	
C 3765....	Apr. 25	14 58	9.527	+37.7	+ 4	1	
C 3767....	Apr. 25	16 03	9.572	+29.4	− 1	1	

(1) Short camera.
(2) Poor focus.
(3) Not used in forming the normal places.
(4) Poor plate.

would seriously upset the assemblage of velocities on the very steep and therefore critical branch of the curve. This element has been assumed to be known.

Because of the uncertainties already alluded to which may affect individual radial velocity determinations, it was deemed best to use normal places for the derivation of the preliminary elements and their subsequent correction by the method of least-squares. The

TABLE VI

NORMAL PLACES FOR ϵ HYDRAE C

No.	Limits of Phase	Phase	Velocity	O−C	Wt.	Plates Included in Normal Place
			km/sec.	km/sec.		
1	....	$0^d.103$	− 5.8	−0.5	½	γ 11456
2	$0^d.019$–$0^d.331$	.163	− 3.8	+5.1	2	γ 13382, 13430
3	....	.203	−16.3	−6.0	1	γ 11748
4	....	0.438	−14.3	−1.5	1	γ 14126
5	....	1.038	−10.0	−8.7	1	γ 11463
6	1.031–1.243	1.137	+ 2.7	+1.6	2	γ 13174, 14014
7	1.663–1.878	1.771	+ 8.0	−3.0	2	γ 11655, 12449
8	2.153–3.172	2.647	+22.1	+1.0	6	C2236, γ 7946, 11625, 11759, 12407, 13417
9	3.486–4.154	3.859	+26.3	−4.3	3	γ 8038, 11766, 12565
10	4.407–5.459	5.123	+41.4	+3.1	4	γ 5573, 11576, 11772, C 2546
11	5.817–5.962	5.865	+46.5	+4.0	1½	C2643, γ 5361
12	7.201–7.884	7.603	+48.0	−3.7	3	γ 7771, 11645, 12367, C2606
13	....	8.959	+52.1	−3.1	1	γ 13996
14	....	9.128	+56.7	+2.9	1	γ 5419
15	9.361–9.416	9.388	+54.8	+7.2	2	C2280, γ 12304
16	9.527–9.572	9.550	+33.6	−5.7	2	C3765, C3767
17	9.768–0.033	9.881	+ 8.1	+1.0	3	γ 14002, 14006, C3677

data for the forty-four plates are in Table V; the normal places are given in Table VI. Notes to the tables indicate that a few of the spectrograms were not used for the normal places and give the reasons for rejection. Individual observations with phases such that they cover an essentially linear portion of the velocity-curve have been combined to form a normal place. Although there are objections to combining into one normal place observations separated by many periods, that liberty has been taken because the period was already rather well determined.

The preliminary elements used are the mean of sets obtained by the methods of Russell and Lehmann-Filhés. These have been

once corrected by least-squares, which yielded appreciable corrections only in the case of the velocity of the center of mass, γ, and the semi-amplitude of velocity variation, K. The preliminary elements, corrections, and final elements are in Table VII.

The eccentricity, 0.62, is among the very large values for spectroscopic binaries, and is much like the value 0.65 found for both the visual and spectroscopic orbits of components A and B.

If the plane of the orbit of C around AB were normal to the line of sight, we should expect the value of γ for the spectroscopic binary C to be the same as that found by Aitken for AB, provided neither system contained a third body. The velocity of the center of mass

TABLE VII

ELEMENTS FOR ϵ HYDRAE C

	Preliminary Elements	Corrections	Final Elements
P......	....	....	9^d9047
e.......	0.62	0.000	0.62
ω.......	$117^\circ7$	$-0^\circ1$	$117^\circ6$
K......	36.2	-1.2	35.0 km/sec.
T......	J.D. 2423800.000	$+0.007$	3800.007
γ.......	$+30.0$	$+1.2$	$+31.2$ km/sec.
$a \sin i$...	....	....	3,700,000 km
$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	....	....	0.0206 ⊙

for C is, however, about 6 km/sec. less than that derived for AB. Campbell's K-term cannot be responsible, since the two spectra are too nearly alike. There may be a third body in either system; to decide this would mean a great deal more observing. But the orbit of C around AB may not be normal to the line of sight, and the velocity difference of 6 km/sec. may be the component of the orbital velocity of C around AB. Assuming this to be the case, and that the observed difference is the component at the descending node, and further, that the angular separation of C from AB converted into linear measure by the parallax is the projected radius of a circular orbit, we find that the period of C about AB would be of the order of six hundred years. The angular rate of revolution of C about AB thus far observed gives six hundred and fifty years as the period, on the assumption of a circular orbit. Hence the difference in the veloci-

ties of the center of mass for C and AB is compatible with the known motion in position angle.

It may be well to point out that all but the first six observations of the radial velocity of C have been made at times when the position angles of B referred to A were in the same quadrant as those of C referred to AB. Further, since aphelion in the system AB occurs in that quadrant, the configuration mentioned persists longer than for B in any one of the other three quadrants. It would be of interest to redetermine the orbit for star C when star B is in the quadrant opposite to C.

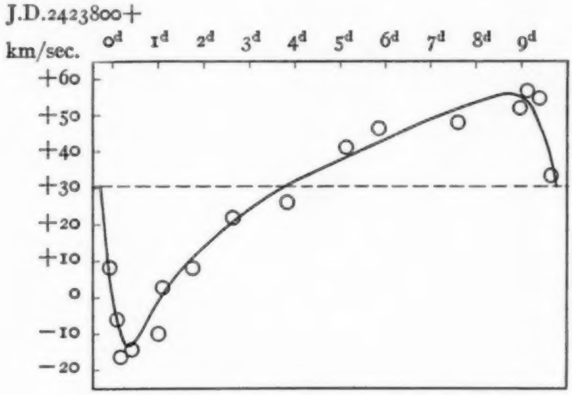

FIG. 2.—Radial velocity curve of ε Hydrae C

Table V gives the residuals for the individual observations as scaled from the velocity-curve based upon the final elements. Table VI shows the differences O−C between the velocities for the normal places and those computed by the final elements. When the practical difficulties are considered, the representation in both tables is quite as satisfactory as could be expected. In Figure 2 the normal places only have been plotted with the computed velocity-curve.

Besides stars A, B, and C, there is a fourth star, D, 19″ distant and of 12.5± mag., which has the same proper motion as the brighter stars and is therefore presumably a member of the system. Since its absolute magnitude is some 8 mag. fainter than that of star A, or about $M=9$, it would be of interest to know something of its spec-

trum. It is hoped that a spectrogram may be obtained in the near future with a spectrograph of low dispersion.

TABLE VIII

OBSERVATIONS OF BOSS 4247

Plate No.	Date	G.M.T.	Phase	Velocity		O−C	
				Prim.	Second.	Prim.	Second.
				km/sec.	km/sec.	km/sec.	km/sec.
γ 11848....	1923 June 6	19^h46^m	0^d108	+ 84.0	−105.5	+ 3.3	+11.1
γ 11859....	June 21	21 11	1.323	− 98.7	+ 95.9	+ 1.2	+10.9
γ 11899....	June 28	18 04	1.270	− 99.6	+ 87.5	+ 5.6	− 3.2
γ 11905....	June 29	19 10	0.008	+ 88.0	−122.2	+ 3.2	− 0.8
γ 12004....	Aug. 5	15 20	2.234	+ 81.9	−132.6	− 1.0	−13.5
γ 12087....	Sept. 1	14 52	1.524	− 72.0	+ 50.2	− 7.5	+ 4.8
C 2425....	Sept. 2	14 57	0.219	+ 67.2	−116.2	− 0.7	−13.6
C 2431....	Sept. 4	14 51	2.215	+ 82.0	−113.4	+ 0.3	+ 4.5
C 2432....	Sept. 4	15 05	2.224	+ 81.8	−116.4	− 0.5	+ 2.1
γ 12534....	1924 Mar. 18	22 30	0.080	+ 81.3	−117.9	− 1.2	+ 0.9
γ 12541....	Mar. 19	22 20	1.073	−108.4	+ 82.0	− 0.8	−11.4
γ 12564....	Apr. 15	21 49	0.360	+ 54.0	− 71.8	+12.4	+ 1.3
γ 12569....	Apr. 16	21 20	1.340	− 93.1	+ 93.0	+ 4.7	+10.0
γ 12575....	Apr. 17	21 58	0.058	+ 76.2	−112.8	− 6.3	+ 7.2
C 2753....	Apr. 18	22 08	1.065	−101.7	+ 97.2	+ 5.5	+ 4.2
γ 12640....	May 15	18 39	0.229	+ 58.6	− 98.6	− 7.8	+ 2.3
γ 12646....	May 16	19 26	1.262	−112.6	+ 90.7	− 6.8	− 0.7
γ 12651....	May 17	20 05	2.289	+ 80.6	−118.9	− 4.1	+ 2.3
C 2785....	May 18	22 46	1.093	−109.7	+ 98.0	− 1.1	+ 3.4
γ 12664....	May 19	22 24	2.077	+ 68.0	−102.8	+ 1.8	− 2.2
γ 12738....	June 17	19 50	0.973	− 94.8	+ 85.2	+ 3.7	+ 1.9
γ 12743....	June 18	17 25	1.872	+ 16.8	....	− 7.2	....
γ 12782....	July 13	15 42	1.416	− 95.2	+ 56.4	− 9.0	−13.2
γ 12789....	July 14	15 52	0.116	+ 84.6	−111.4	+ 4.7	+ 4.5
γ 12795....	July 15	16 35	1.145	−112.9	+ 95.6	− 2.9	− 0.6
C 3313....	1925 June 5	18 15	0.842	− 71.8	+ 63.7	+ 5.2	+ 4.3
γ 13438....	June 7	17 32	0.505	− 5.4	....	−11.7	....
γ 13444....	June 7	23 26	0.750	− 67.7	+ 41.3	−10.9	+ 4.5
γ 13445....	June 8	15 30	1.420	− 86.5	+ 61.5	− 0.9	− 7.4
γ 13450....	June 8	23 28	1.752	− 13.8	....	− 6.8	....

ORBITS OF THE TWO COMPONENTS OF THE SPECTROSCOPIC BINARY BOSS 4247

The first spectrogram of this star showed the lines of two stars, both of spectral class about F2, and established its binary character.

Thirty spectrograms with one-prism dispersion were made with the 60- and 100-inch reflectors between June, 1923, and June, 1925. The data for these plates are in Table VIII. Both the phases and the residuals refer to the adopted elements. The period, which was

soon found to be near 2 days, turned out to be 2.3076 days, and satisfactorily represents all observations in an interval which includes about three hundred and thirty revolutions in the orbit. The distribution of velocities would be very seriously affected were this period changed as much as 0.0005 day, which may be taken as an upper limit of the possible error in P. A freehand velocity-curve through the assembled observations was not distinguishable from a sine curve, and the orbit has been assumed to be circular. The elements are therefore period, velocity of the system, semi-amplitude of velocity variation for both components, and an arbitrarily

TABLE IX

ELEMENTS FOR BOSS 4247

	Preliminary Elements	Corrections	Corrected Elements
P	....	....	2^d3076
K	97.5	−0.11	97.4 km/sec.
K_1	107.5	+1.22	108.7 km/sec.
T	J.D. 2423923.862	−0.006	3923.856
γ	−12.5	−0.09	−12.6 km/sec.
$(a+a_1)\sin i$	....	....	6,540,000 km
$m \sin^3 i$	....	....	1.1065 ⊙
$m_1 \sin^3 i$	....	....	0.9915 ⊙
$\frac{m}{m_1}=\frac{K^1}{K}$	....	....	1.11

chosen epoch, in this case the time when the primary attains its maximum positive velocity. These were determined by the graphical method of Lehmann-Filhés, and once corrected by the method of least-squares. Table IX gives these elements, the corrections, and the corrected elements which have been adopted. The representation is quite satisfactory as attested by the residuals in Table VIII and the fit of the observed velocities to the curve as shown in Figure 3.

The values of $a \sin i$ and $a_1 \sin i$ are 3,114,300 and 3,425,700 km, respectively. If the radii of the two stars are of the order of that of the sun, any inclination between 70° and 90° will result in a partial eclipse twice each revolution at phases corresponding to the intersections of the two curves in the radial velocity diagram. It might prove of interest to test this point by photometric observations.

Measures of the trigonometric parallax of this system are un-

known to me, and the determination of the absolute magnitudes by spectral criteria is made difficult on account of the presence of the two sets of lines.

Boss's proper motion is small. The system appears, however, to consist of two stars not greatly different in absolute magnitude from that of the sun.

B.D. $+57^\circ 2309$

Barnard found this star to be involved in nebulosity, as may be seen by reference to Plate 82, *Publications of the Lick Observatory*, **11**,

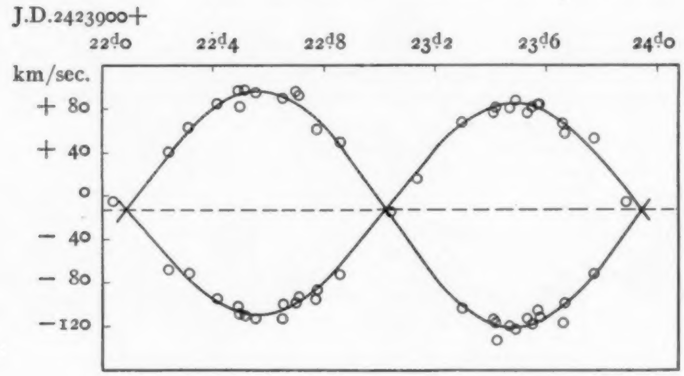

FIG. 3.—Radial velocity curves of Boss 4247

1913, where it appears as the first object listed in the appended table. This led Hubble, in 1920, to observe its spectrum with the one-prism spectrograph and 60-inch reflector. His five plates gave certain evidence of variability in the radial velocity of the star, and velocities from the H and K lines of calcium which did not agree with those from the other lines. He noted that $H\beta$ was an abnormally weak absorption line, and found from objective-prism plates that $H\alpha$ appeared as an emission line.[1] The character of $H\alpha$ has been verified by slit spectrograms.

The star was placed on the program of spectroscopic binaries in 1921, and by the end of 1925 forty-two usable spectrograms had been obtained. These have furnished the data for the investigation

[1] Adams, Joy, and Sanford, *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

of the orbit. Considerable difficulty was encountered in deriving a period until it was noted that observations which evidently established maxima and minima could not be harmonized with a single

TABLE X

NORMAL PLACES; B.D.+57°2309; $P=225^{d}.44$

No.	Julian Day	Phase	Velocity Deviation from 5-Day Orbit	O−C	Wt.	Nos. from Column 1, Table XII
			km/sec.	km/sec.		
1	2424490.13	$9^{d}.73$	+19	+ 4	2	40, 41
2	3599.42	20.88	+ 4	− 3	4	10, 11, 12, 13
3	4514.61	34.31	− 8	− 4	1	42
4	4310.45	55.59	−23	− 7	2	30, 31
5	3636.63	58.10	−11	+ 6	3	14, 15, 16
6	4339.67	84.81	−19	+ 4	6	32, 33, 34, 35, 36, 37
7	3665.82	87.29	−27	− 4	5	17, 18, 19, 20, 21
8	3014.72	112.50	−31	− 9	1	6
9	3696.80	118.26	−34	−12	1	22
10	3923.97	119.99	−22	− 1	3	27, 28, 29
11	3720.73	142.19	−10	+ 6	4	23, 24, 25, 26
12	2609.45	158.11	−12	− 2	4	1, 2, 3, 4
13	4420.18	165.32	−11	− 4	2	38, 39
14	3570.64	217.54	+22	+ 1	3	7, 8, 9

TABLE XI

B.D.+57°2309—ELEMENTS OF LONG-PERIOD VARIATION

	Preliminary Elements	Corrections	Adopted Elements
P	....	....	$225^{d}.44$
e	0.095	+ 0.131	0.226
ω	0°	+16°.6	16°.6
K	26.4	− 4.5	21.9 km/sec.
T	J.D. 2423797.50	+ 6.48	3803.98
γ	−4.5	− 1.6	−6.1 km/sec.
$a \sin i$	....	....	65,988,600 km
$\frac{M_1^3 \sin^3 i}{(M+M_1)^2}$	....	....	0.3956 ⊙

constant period. An attempt to establish a periodicity for these changes indicated a cycle of about 225 days. Further observation seemed to confirm this, so that a tentative curve having a period of 225 days was used as a starting-point for the correction of the observed velocities.

When the corrected velocities were studied, it was found that $P = 5^{d}.41364$ provided an assembly which, in view of the nature of the spectrum from which the velocities were obtained, was fairly satisfactory. As the lines of only one star are visible, and their breadth and diffuseness make them somewhat difficult to measure, the velocities are subject to rather large probable errors. The calcium lines H and K are sharp and fairly strong, however, and special reference will be made to them later. The proximity to $H\epsilon$ makes the H line less satisfactory than the K line.

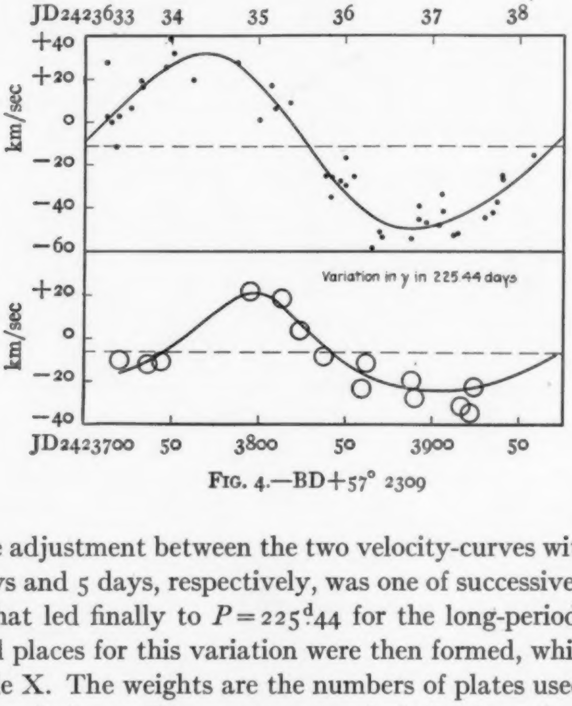

FIG. 4.—BD+57° 2309

The adjustment between the two velocity-curves with periods of 225 days and 5 days, respectively, was one of successive approximations that led finally to $P = 225^{d}.44$ for the long-period oscillation. Normal places for this variation were then formed, which are given in Table X. The weights are the numbers of plates used in forming the normal places. The spectrograms included in each normal place are indicated by the numbers in the last column, which refer to the successive plates in Table XII. The velocity deviations in Table X are residuals from the best approximation that could be gotten for the short-period velocity-curve. The velocity-curve which results from a plot of these data was used to derive preliminary elements by the method of Lehmann-Filhés. One least-squares solution for

TABLE XII

OBSERVATIONS OF B.D.+57°2309

No.	Plate No.	Date	G.M.T.	Phase	Obs. Vel.	Corr. from Long-Per. Var.	Corr. Vel.	O−C	H and K Vel.
		1920			km/sec.	km/sec.	km/sec.	km/sec.	km/sec.
1....	γ 9609	Oct. 2	18h27m	5d048	+13.4	+14.5	+27.9	+ 1	......
2....	γ 9619	Oct. 3	18 29	0.635	−39.2	+14.0	−25.2	− 3	......
3....	γ 9626	Oct. 20	14 58	1.248	−57.6	+ 6.5	−51.1	− 5	......
4....	γ 9627	Oct. 20	15 45	1.280	−60.3	+ 6.5	−53.8	− 7	......
5....	γ 9717	Nov. 19	17 46	4.296	+42.3	−10.5	+31.8	+ 5	......
		1921							
6....	γ 10651	Nov. 20	17 21	2.152	−75.0	+23.0	−52.0	− 8	......
		1923							
7....	γ 11814	May 28	23 30	4.210	+45.6	−20.0	+25.6	0	......
8....	C 2279	May 30	23 15	0.793	− 7.4	−20.0	−27.4	+ 3	......
9....	C 2283	May 31	23 07	1.787	−26.8	−20.0	−46.8	+ 2	−15.7
10....	C 2325	June 26	22 18	0.685	−26.6	− 8.5	−35.1	−10	−13.9
11....	C 2330	June 27	22 30	1.693	−36.8	− 8.5	−45.3	+ 4	......
12....	γ 11903	June 28	21 51	2.665	−18.8	− 6.0	−24.8	+ 6	......
13....	γ 11908	June 29	21 14	3.569	+ 6.0	− 6.0	0.0	− 3	−11.8
14....	γ 11996	Aug. 3	20 16	0.704	−41.8	+16.0	−25.8	0	−11.2
15....	γ 12002	Aug. 4	20 05	1.707	−54.9	+16.0	−38.9	+10	......
16....	γ 12007	Aug. 5	19 29	2.672	−43.7	+17.0	−26.7	+ 5	......
17....	γ 12090	Sept. 1	18 15	2.552	−65.6	+23.5	−42.1	− 7	......
18....	γ 12091	Sept. 1	19 13	2.593	−60.6	+23.5	−37.1	− 4	......
19....	C 2428	Sept. 2	19 56	3.622	−35.5	+23.5	−12.0	−16	*
20....	C 2429	Sept. 2	20 42	3.655	−21.1	+23.5	+ 2.4	− 3	−14.2
21....	C 2435	Sept. 4	20 27	0.230	−14.2	+23.5	+ 9.3	+ 8	−16.1
22....	C 2478	Oct. 3	19 5	2.105	−74.6	+22.0	−52.6	− 7	−15.5
23....	C 2493	Oct. 21	17 40	3.805	−11.2	+18.0	+ 6.8	− 5	−28.5
24....	γ 12245	Oct. 28	18 20	0.006	+ 1.3	+16.0	+17.3	+ 6	+ 4.2
25....	γ 12249	Oct. 29	16 44	0.939	−41.5	+16.0	−25.5	+11	−18.0
26....	γ 12257	Oct. 30	16 59	1.950	−49.3	+15.5	−33.8	+12	−38.6
		1924							
27....	γ 12649	May 16	23 31	0.856	−51.6	+22.0	−29.6	+ 3	......
28....	γ 12653	May 17	23 32	1.918	−70.0	+22.0	−48.0	− 1	......
29....	C 2792	May 19	23 56	3.934	− 4.5	+21.5	+17.0	0	−16.6
		1925							
30....	γ 13443	June 7	22 52	3.522	−13.0	+15.5	+ 2.5	+ 2	+ 2.3
31....	γ 13449	June 8	22 50	4.520	+ 3.8	+16.0	+19.8	−11	......
32....	γ 13518	July 5	18 18	4.263	+15.9	+23.0	+38.9	+12	......
33....	γ 13524	July 6	18 28	5.270	−22.2	+23.0	+ 0.8	−16	......
34....	γ 13527	July 6	22 50	0.038	−16.5	+23.0	+ 6.5	− 3	......
35....	γ 13531	July 7	17 46	0.827	−40.4	+23.5	−16.9	+16	......
36....	γ 13539	July 8	21 18	1.974	−65.1	+23.5	−41.6	+ 5	......
37....	γ 13545	July 9	22 14	3.013	−38.6	+23.5	−15.1	+ 4	......
38....	γ 13738	Sept. 24	14 56	3.918	+11.1	+ 8.0	+19.1	+ 3	......
39....	γ 13760	Sept. 27	17 39	1.617	−60.8	+ 6.5	−54.3	− 5	......
40....	C 3588	Dec. 3	14 42	3.532	+43.8	−16.0	+27.8	+27	−34.4
41....	γ 13912	Dec. 6	15 20	1.144	−44.6	−14.5	−59.1	−15	......
42....	γ 13942	Dec. 29	14 34	2.458	−46.8	+ 2.5	−44.3	− 7	......

* Red sensitive plate.

corrections to these elements was made, and Table XI gives preliminary elements, corrections, and corrected elements. The element called γ in this case must be considered as a correction to the γ corresponding to the short-period orbit, so that the velocity of the triple system is the sum of these two gammas.

The lower part of Figure 4 shows the curve defined by the corrected elements of Table XI and the normal points from which it was derived. It is doubtful whether these elements are quite right, but the limited data hardly warrant further refinement.

TABLE XIII

NORMAL PLACES; B.D.+57°2309; $P=5^{\mathrm{d}}.41364$

No.	Phase	Velocity Freed from Long-Period Oscillation	O−C	Wt.	Plate Nos. from Table XII
		km/sec.	km/sec.		
1........	$0^{\mathrm{d}}.118$	+13.3	+ 7	2	21, 24
2........	0.795	−28.7	+ 2	5	8, 10, 14, 25, 27
3........	0.986	−38.0	− 1	2	35, 41
4........	1.054	−43.4	− 3	3	2, 3, 4
5........	1.796	−48.0	0	2	36, 39
6........	1.860	−44.2	+ 4	6	9, 11, 15, 22, 26, 28
7........	2.152	−52.0	− 8	1	6
8........	2.620	−32.7	0	4	12, 16, 17, 18
9........	2.740	−29.7	− 1	2	37, 42
10........	3.693	− 0.7	− 8	4	13, 19, 20, 23
11........	3.758	+ 9.8	0	2	29, 30
12........	4.088	+26.4	+ 6	4	31, 32, 38, 40
13........	4.548	+28.4	− 2	3	1, 5, 7
14........	5.391	+ 3.6	−10	2	33, 34

The lower curve in Figure 4 was then used to derive the corrections to the observed velocities to free them from the superposed long-period variation. Table XII gives the various data for the forty-two plates. The phases depend on the finally adopted elements of the short-period orbit; the corrections in the sixth column are those just referred to. The residuals are scaled from the upper part of Figure 4.

The assembly of the corrected velocities based on $P=5^{\mathrm{d}}.41364$ was so satisfactory that the period could be assumed to be known.

On account of the rather low weight for a single observation, it seemed best to form normal places, a procedure which also greatly

diminished the labor of correcting the preliminary elements. These normal places are given in Table XIII. The weights are equal to the numbers of plates forming a normal. The phase and residuals refer to the final elements.

The freehand curve drawn through the plot of the normal places of Table XIII, assembled with the period 5.41364 days, gave by Russell's method a set of preliminary elements, which, as in the other case, were once corrected by least-squares. Table XIV gives the preliminary elements, corrections, and corrected elements. In this

TABLE XIV

B.D.+57°2309—ELEMENTS OF SHORT-PERIOD VARIATION

	Preliminary Elements	Corrections	Adopted Elements
P			5^d41364
e	0.10	+0.014	0.114
ω	63°5	−5°0	58°5
K	41.3	−1.3	40.0 km/sec.
T	J.D. 2423635.137	+0.003	3635.140
γ	−11.3	+0.2	−11.1 km/sec.
$a \sin i$			2,959,900 km
$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$			0.0353 ☉

case, the corrections are satisfactorily small. By combining the γ thus derived with that from Table XI, the velocity of the triple system is found to be −17.2 km/sec.

The values for $a \sin i$ and the mass function have both been added to Tables XI and XIV. In the former M_1 is the mass of the third body, and a and K refer to the semi-major axis and projected semi-amplitude of the velocity variation of the pair of stars forming the short-period system about the center of gravity of the triple system; and M is the combined mass of the two stars forming the short-period system and therefore the sum of m and m_1 which appear in Table XIV. If we assume that the mass of the primary in the short-period system is twice that of the secondary and that i, the inclination of the orbit plane to the plane normal to the line of sight, is the same for both orbits, it is readily shown that we are dealing with moderate and entirely reasonable masses, whatever the value of i between 30° and 90°. Further discussion of this point

is hardly warranted, since the assumptions have no observational foundation.

The last column of Table XII gives the velocities derived from the H and K lines of calcium whenever these lines have been of satisfactory quality for measurement. There is considerable range in these velocities; but it has not been possible to find any smooth variation based on either the long or short period or a combination of the two. This does not preclude the possibility of periodicity, but the present data can be just as well represented by an invariable velocity whose apparent variability is only accidental. Hence the weighted mean is used as the best value of the velocity based upon the H and K lines, and this will probably not be far from the truth even if there is a periodic variation. This mean is -19.4 km/sec., which is not greatly different from the velocity of the triple system, -17.2 km/sec., or from the reflex of the solar motion in the direction of this system, namely, -15 km/sec. If it may be identified with the velocity of the triple system, which seems reasonable, and if the source of the H and K absorption is assumed to be in the immediate vicinity of the system, which is closely surrounded by nebulosity, it would be natural to infer that the velocity of the triple system is also that of the nebulosity, and that such nebulosity has ionized calcium as one of its constituents. So far as known, the spectrum of the nebulosity is continuous in character. The sodium lines D_1 and D_2 are present, a point of interest, because, except when these two lines manifest a behavior similar to the detached H and K lines, they do not ordinarily appear until later in the stellar spectral sequence.

Since, as already mentioned in connection with η Orionis A, several spectroscopic binaries which have detached H and K lines also have detached sodium lines, a spectrogram covering the D region of B.D. $+57^\circ 2309$ was made on June 22, 1926. The photographic density is such that the spectrum can be measured from K of calcium to the D lines. The strength of the D lines is in itself sufficient to show that these lines are of the detached variety, since, otherwise, they would be almost vanishingly weak, inasmuch as the spectrum of the star is B3. The measured velocities also support this conclusion. The epoch was so chosen that the star's velocity was at

the maximum for both the short and long periods, and hence deviated a maximum amount from the velocity of the triple system. The velocities for the K line and for the D lines and the mean velocity from the remaining lines are -2, -10, and $+60$ km/sec., respectively. The third velocity is to be compared with a computed velocity of $+50$ km/sec. The D-line velocity certainly differs markedly from this value, and is of the order of the previously determined mean for H and K and of the value for the single K line on the same plate. The evidence from this single plate is therefore for a similarity in behavior between the sodium lines and the H and K lines. In order to settle the question definitely, however, more plates are needed, especially such as have a greater precision for the D lines.

MOUNT WILSON OBSERVATORY
July 1926

MULTIPLETS IN THE SPARK SPECTRUM OF IRON[1]

By HENRY NORRIS RUSSELL

ABSTRACT

Analysis of the spark spectrum of iron.—All the lines of astrophysical importance have been classified with the aid of material prepared by Messrs. Merrill, Anderson, and Babcock. *Multiplets* belonging to *sextet and quartet systems* have been found. So far 214 lines have been grouped into 35 multiplets, involving combinations between 16 terms.

The *state of lowest energy* in the ionized atom corresponds to a ^{6}D-term, the next to a ^{4}F′-term. *The observed terms are in excellent agreement with the predictions of Hund's theory,* and are from the electronic configurations s^2d^5, sd^6, d^7, and d^6p. Only the lowest terms of the quartet system out of a large number predicted have yet been found, and an intricate system of doublets should exist. This agrees with the fact that many lines are still unclassified. Work on this spectrum will be continued.

An investigation of the enhanced lines of iron has been planned at this Observatory for some time. In connection with this, a list of all the iron lines which are known to belong to the spark spectrum was prepared by Mr. Merrill and Mr. Anderson, including the Zeeman effects as measured by Mr. Babcock, and used by the writer as a basis for an analysis of the structure of the spectrum.

The starting-point of this analysis was found in two multiplets in the ultra-violet discovered at the Bureau of Standards[2] which arose from combinations between a D- and D′-term of a sextet system (^{6}D and ^{6}D′), and of a ^{4}D- and ^{4}P-term of a quartet system. With these as a beginning, a considerable number of other terms was found, and practically all the enhanced lines in the visible and near ultra-violet region were classified. A summary of the results appears in the writer's "List of Ultimate and Penultimate Lines,"[3] but the details have not yet been published.

Many lines remain unassigned, and it is hoped to continue the investigation in the near future; but as practically all the lines of astrophysical importance have already been classified, and there can be little doubt that the principal low energy-levels have been found,

[1] *Contributions from the Mount Wilson Observatory*, No. 318.

[2] Meggers, Kiess, and Walters, *Journal of the Optical Society of America*, **9**, 371, 1924.

[3] *Mt. Wilson Contr.*, No. 286; *Astrophysical Journal*, **61**, 274, 1925.

it seems desirable to publish the results thus far obtained, for the benefit of those who may be interested.

The lowest energy-level of the ionized atom of iron belongs to a ^{6}D-term, which, with all the others so far found, is "inverted," the component with the greatest inner-quantum number having the lowest energy. Next above this and very near it comes a ^{4}F′-term, and then a ^{4}D- and a ^{4}P′-term. These combine with terms of higher levels of types ^{6}P ^{6}D′ ^{6}F, ^{4}P ^{4}D′ ^{4}F, giving strong lines in the ultra-violet, which appear with considerable and often great intensity in the arc spectrum.

The spark lines in the visible region arise from the combination of higher-lying quartet and sextet terms with the same two "triads" of terms which have just been mentioned, as is shown in detail in the tables.

Table I gives the spectroscopic terms which have so far been identified. The notation is that now in general use, $^4P_3$ for example, denoting that component of a P-term of the quartet system which has the inner quantum number 3. Different terms of the same species are distinguished by letters a^{4}P, b^{4}P, etc., "a" denoting the lowest known term of the sort. The term values are counted upward from the lowest known energy-level, which appears to be much the best scheme when the ionization potential is unknown, and has much to recommend it as a permanent policy in the case of complex spectra.

A list of the lines which have been identified is given in Table II, which includes wave-lengths in I.A., intensities in the spark (from Merrill and Anderson's list), wave-numbers, and the designation by means of the terms whose combination produces the line.

The third column (O−C) gives the difference between the observed wave-length and that computed from the term values given in Table I. The agreement is very close, except for the lines in the extreme ultra-violet, which are taken from Bloch's measures.[1] There is a systematic discordance of 0.10 A in this case, which, in view of the difficulties of exact standardization of measures of wave-length in this region, may be regarded as remarkably small.

When more such groups of lines of predictable wave-lengths have

[1] *Comptes Rendus*, **179**, 1396, 1924.

TABLE I

Fe+ TERMS

Type	Term	Combinations
$a^4P'_3$	13474.36	a^4P a^4D' a^4F
	198.68	
$a^4P'_2$	13673.04	
	231.70	
$a^4P'_1$	13904.74	
$b^4P'_3$	20830.44	a^4P a^4D' a^4F
	981.49	
$b^4P'_2$	21811.93	
	597.78	
$b^4P'_1$	22409.71	
a^4D_4	7955.24	a^4P a^4D' a^4F
	436.66	
a^4D_3	8391.90	a^6P a^6D' a^6F
	288.47	
a^4D_2	8680.37	
	166.35	
a^4D_1	8846.72	
b^4D_4	31483.10	a^4P a^4D' a^4F
	−95.28	
b^4D_3	31387.82	b^4F
	−23.51	
b^4D_2	31364.31	
	3.99	
b^4D_1	31368.30	
$a^4F'_5$	1872.56	a^4D' a^4F b^4F
	557.60	
$a^4F'_4$	2430.16	a^6D' a^6F
	407.75	
$a^4F'_3$	2837.91	
	279.58	
$a^4F'_2$	3117.49	
$b^4F'_5$	22637.15	a^4D' a^4F b^4F
	173.13	
$b^4F'_4$	22810.28	
	128.93	
$b^4F'_3$	22939.21	
	92.04	
$b^4F'_2$	23031.25	
a^4G_6	25428.78	a^4D' a^4F b^4F
	376.43	
a^4G_5	25805.21	
	176.30	
a^4G_4	25981.51	
	73.84	
a^4G_3	26055.35	

Type	Term	Combinations
a^6S	23317.61	a^6P a^6D' a^6F a^4D' a^4F
a^6D_5	0.00	a^6P a^6D' a^6F
	384.80	
a^6D_4	384.80	
	282.85	
a^6D_3	667.65	
	194.90	
a^6D_2	862.55	
	114.41	
a^6D_1	976.96	
a^4P_3	46967.29	a^4P' b^4P' a^4D
	422.34	
a^4P_2	47389.63	b^4D
	236.29	
a^4P_1	47625.92	
$a^4D'_4$	44446.81	a^4P' b^4P' a^4D
	337.81	
$a^4D'_3$	44784.62	b^4D a^4F' b^4F'
	259.53	
$a^4D'_2$	45044.15	a^4G a^6S
	162.23	
$a^4D'_1$	45206.38	
a^4F_5	44232.52	a^4P' b^4P' a^4D
	521.20	
a^4F_4	44753.72	b^4D a^4F' b^4F'
	326.05	
a^4F_3	45079.77	a^4G a^6S
	209.99	
a^4F_2	45289.76	
b^4F_5	65695.28	b^4D a^4F' b^4F'
	681.63	
b^4F_4	66376.91	a^4G
	453.41	
b^4F_3	66830.32	
	234.54	
b^4F_2	67064.86	
a^6P_4	42658.26	a^6S a^6D a^4D
	580.26	
a^6P_3	43238.52	
	382.46	
a^6P_2	43620.98	

TABLE I—*Continued*

Type	Term	Combinations	Type	Term	Combinations
$a^6D'_5$	38458.88	a^6S a^6D a^4D	a^6F_6	41968.05	a^6S a^6D a^4D
	201.05			*146.78*	
$a^6D'_4$	38659.93	a^4F'	a^6F_5	42114.83	a^4F'
	198.91			*122.17*	
$a^6D'_3$	38858.84		a^6F_4	42237.00	
	154.19			*97.62*	
$a^6D'_2$	39013.03		a^6F_3	42334.62	
	96.10			*66.58*	
$a^6D'_1$	39109.13		a^6F_2	42401.20	
				38.51	
			a^6F_1	42439.71	

been found, the problem of obtaining accurate standards in the Schumann region will be practically solved.

A number of lines in the red which fall exactly in the positions predicted by the combination principle, but have not been recorded in the laboratory, have been taken from the solar spectrum, in which they are readily distinguished by the fact that they are greatly weakened in sun-spots. These lines are indicated by an asterisk (*) in the intensity column. They are all faint. This behavior is characteristic of enhanced lines and suffices to show that they belong to a spark spectrum of some sort. The excellent agreement with the computed wave-lengths indicates that they are due to iron.

A few words may be added on the theoretical significance of these results.

According to Hund's theory,[1] which gives a remarkably complete account of the structure of all spectra to which it has been applied, the more complex spectra arise from the combined action of all the "outer" electrons on the atom (those which do not belong to complete groups or "shells"). Each of these electrons may be described in the usual way by the quantum number of its orbit, e.g., 4_1, 4_2, 3_3, where the large figure denotes the total and the subscript the azimuthal quantum number, or by the equivalent notation 4s, 4p, 3d, which describes the type of spectroscopic term which would be produced by such an electron if acting alone (as in the cases of K, Ca^+, etc.).

To a given configuration of electronic orbits in the atom corre-

[1] *Zeitschrift für Physik*, **33**, 345, 1925.

TABLE II

Fe^+ Identified Lines

(Unit for O−C, 0.01 A)

λ	Int.	O−C λ	ν Vac.	Multiplet
1550.17	3	−12	64509.07	$a^4F'_5-b^4F_4$
1552.7	1	− 9	64403.95	$a^4F'_4-b^4F_3$
1562.60	00	− 9	63995.93	$a^4F'_3-b^4F_3$
1563.70†	3	{−10, − 9}	63950.87	{$a^4F'_4-b^4F_4$, $a^4F'_2-b^4F_2$}
1566.74	2	−10	63826.81	$a^4F'_5-b^4F_5$
1580.53	2	−12	63269.93	$a^4F'_4-b^4F_5$
2270.29	1	0	44033.61	$b^4F'_2-b^4F_2$
2277.67	4	+ 1	43890.91	$b^4F'_3-b^4F_3$
2294.69	5	+ 1	43565.42	$b^4F'_4-b^4F_4$?
2321.71	1	− 2	43058.46	$b^4F'_5-b^4F_5$
2327.39	5	0	42953.38	$a^6D_3-a^6P_2$
2331.30	5	− 1	42881.35	$a^4F'_5-a^4F_4$
2332.80	6	0	42853.79	$a^6D_4-a^6P_3$
2338.00	5	0	42758.48	$a^6D_2-a^6P_2$
2343.49	7	0	42658.28	$a^6D_5-a^6P_4$
2343.97	3	+ 4	42648.87	$a^4F'_4-a^4F_3$?
2344.28	5	0	42643.95	$a^6D_1-a^6P_2$
2348.12	6	0	42574.22	$a^4F'_5-a^4D'_4$
2348.30	7	− 1	42570.96	$a^6D_3-a^6P_3$
2354.88	3	− 1	42452.00	$a^4F'_3-a^4F_2$
2359.11	6	0	42375.90	$a^6D_2-a^6P_3$
2360.00	5	0	42359.91	$a^4F'_5-a^4F_5$
2360.31	5	+ 1	42354.36	$a^4F'_4-a^4D'_3$
2362.04	4	+ 1	42323.33	$a^4F'_4-a^4F_4$
2364.82	6	− 1	42273.59	$a^6D_4-a^6P_4$
2366.59	4	− 1	42241.97	$a^4F'_3-a^4F_3$
2366.86	0	+ 7	42235.75	$a^6D_5-a^6F_4$?
2368.60	6	+ 1	42206.13	$a^4F'_3-a^4D'_2$
2370.50	3	0	42172.30	$a^4F'_2-a^4F_2$
2373.733	7	0	42114.87	$a^6D_5-a^6F_5$
2375.193	6	− 1	42088.98	$a^4F'_2-a^4D'_1$
2379.276	6	− 1	42016.75	$a^4F'_4-a^4D'_4$
2380.763	6	+ 1	41990.52	$a^6D_3-a^6P_4$
2382.039	10	0	41968.05	$a^6D_5-a^6F_6$
2383.02	3h	0	41950.75	$a^6D_4-a^6F_3$?
2383.253	4	0	41946.68	$a^4F'_3-a^4D'_3$
2384.39	5	0	41926.65	$a^4F'_2-a^4D'_2$
2385.00	2	− 1	41915.93	$a^4F'_3-a^4F_4$
2388.631	8	0	41852.22	$a^6D_4-a^6F_4$
2391.485	3	0	41802.36	$a^4F'_4-a^4F_5$
2395.423	5	0	41733.56	$a^6D_3-a^6F_2$
2395.628	8	0	41729.99	$a^6D_4-a^6F_5$
2399.244†	8	{− 1, 0}	41667.09	{$a^6D_3-a^6F_3$, $a^4F'_2-a^4D'_3$}
2402.60	3	0	41608.92	$a^4F'_3-a^4D'_4$
2404.435	5	0	41577.16	$a^6D_2-a^6F_1$
2404.888	8	0	41569.33	$a^6D_3-a^6F_4$
2406.663	7	0	41538.67	$a^6D_2-a^6F_2$
2410.526	8	+ 1	41471.98	$a^6D_2-a^6F_3$
2411.071	7	0	41462.74	$a^6D_1-a^6F_1$

TABLE II—*Continued*

λ	Int.	O−C λ	ν Vac.	Multiplet
2413.313	6	0	41424.22	$a^6D_1-a^6F_2$
2437.72	1	0	41009.51	$a^4G_3-b^4F_2$
2447.32	3	+1	40848.65	$a^4G_4-b^4F_3$
2451.74	2	0	40775.01	$a^4G_3-b^4F_3$
2464.03	4	0	40571.66	$a^4G_5-b^4F_4$
2474.78	4	0	40395.44	$a^4G_4-b^4F_4$
2476.67§	2	−1	40364.61	$a^4F'_5-a^6F_4$
2482.70	5	0	40266.58	$a^4G_6-b^4F_5$
2484.188§	6	−1	40242.45	$a^4F'_5-a^6F_5$
2493.29	10	0	40095.56	$a^4F'_5-a^6F_6$
2505.26	1	+3	39904.00	$a^4F'_4-a^6F_3$
2506.14	2	+1	39889.99	$a^4G_5-b^4F_5$
2511.36	2	−2	39807.09	$a^4F'_4-a^6F_4$
2519.09	6	−2	39684.93	$a^4F'_4-a^6F_5$
2549.12	4	−2	39217.46	$a^4F'_2-a^6F_3$
2562.543	9	0	39012.05	$a^4D_4-a^4P_3$
2563.485	7	0	38997.72	$a^4D_3-a^4P_2$
2566.921	5	0	38945.54	$a^4D_2-a^4P_1$
2577.930	6	0	38779.20	$a^4D_1-a^4P_1$
2582.590	8	0	38709.25	$a^4D_2-a^4P_2$
2585.886	10	0	38659.91	$a^6D_5-a^6D'_4$
2591.554	7	0	38575.38	$a^4D_3-a^4P_3$
2593.70†	4	−4	38543.46	$a^4D_1-a^4P_2$?
2598.380	8	0	38474.04	$a^6D_4-a^6D'_3$
2599.405	10	0	38458.88	$a^6D_5-a^6D'_5$
2607.099	8	0	38345.39	$a^6D_3-a^6D'_2$
2611.086	3	0	38286.82	$a^4D_2-a^4P_3$
2611.885	9	0	38275.11	$a^6D_4-a^6D'_4$
2613.835	8	0	38246.57	$a^6D_2-a^6D'_1$
2617.627	8	0	38191.17	$a^6D_3-a^6D'_3$
2620.419	3	0	38150.47	$a^6D_2-a^6D'_2$
2621.677	7	0	38132.17	$a^6D_1-a^6D'_1$
2625.676	9	0	38074.08	$a^6D_4-a^6D'_5$
2628.303	8	0	38036.05	$a^6D_1-a^6D'_2$
2631.053	7	0	37996.30	$a^6D_2-a^6D'_3$
2631.332	7	0	37992.27	$a^6D_3-a^6D'_4$
2692.842	3	0	37124.50	$a^4D_4-a^4F_3$
2709.39	1	+2	36897.62	$a^4D_3-a^4F_2$
2714.419	8	0	36829.39	$a^4D_4-a^4D'_3$
2724.892	5	0	36687.85	$a^4D_3-a^4F_3$
2727.542	5	0	36652.21	$a^4D_3-a^4D'_2$
2730.740	6	+1	36609.30	$a^4D_2-a^4F_2$
2732.46	5	0	36586.27	$a^4F_5-a^6D'_5$
2736.970	7	0	36525.96	$a^4D_2-a^4D'_1$
2739.551	10	0	36491.55	$a^4D_4-a^4D'_4$
2743.199	8	0	36443.03	$a^4D_1-a^4F_2$
2746.486	8	0	36399.41	$a^4D_2-a^4F_3$
2746.988	7	0	36392.75	$a^4D_3-a^4D'_3$
2749.182	4	0	36363.74	$a^4D_2-a^4D'_2$
2749.327	10	0	36361.81	$a^4D_3-a^4F_4$
2749.487	3	0	36359.69	$a^4D_1-a^4D'_1$
2755.736	9	0	36277.24	$a^4D_4-a^4F_5$
2761.809	4	0	36197.48	$a^4D_1-a^4D'_2$
2768.938	3	0	36104.28	$a^4D_2-a^4D'_3$

TABLE II—*Continued*

λ	Int.	O−C λ	ν Vac.	Multiplet
2861.20	2	+4	34940.13	$a^4D_2-a^6P_2$?
2864.97	2	−3	34894.16	$b^4D_4-b^4F_4$?
2868.88	2	0	34846.62	$a^4D_3-a^6P_3$
2880.757	5	+1	34702.94	$a^4D_4-a^6P_4$
2892.85	1	0	34558.11	$a^4D_2-a^6P_3$
2916.1	1u	−5	34282.36	$a^4D_4-a^6F_4$
2922.02	2	−6	34212.91	$b^4D_4-b^4F_5$
2926.584	6	0	34159.55	$a^4D_4-a^6F_5$
2944.400	6	0	33952.89	$a^4P'_2-a^4P_1$
2947.661	6	0	33915.23	$a^4P'_3-a^4P_2$
2953.778	5	0	33845.08	$a^4D_3-a^6F_4$
2964.632†	3	3 0	33721.17	$a^4D_2-a^6F_2$ $a^4P'_1-a^4P_1$
2965.040	4	+1	33716.53	$a^4P'_2-a^4P_2$
2970.518	3	−1	33654.37	$a^4D_2-a^6F_3$
2975.94	1	−1	33593.05	$a^4D_1-a^6F_1$
2979.356	2	0	33554.52	$a^4D_1-a^6F_2$
2984.834	8	0	33492.96	$a^4P'_3-a^4P_3$
2985.552	6	0	33484.91	$a^4P'_1-a^4P_2$
3002.651	7	0	33294.22	$a^4P'_2-a^4P_3$
3170.346	2	+1	31533.22	$a^4P'_2-a^4D'_1$
3183.126	3	+1	31406.60	$a^4P'_2-a^4F_3$
3185.33	....	+2	31384.87	$a^4P'_1-a^4F_2$
3186.750	4	+2	31370.88	$a^4P'_2-a^4D'_2$
3192.806‖	3	−12	31311.38	$a^4P'_3-a^4D'_3$
3193.811	4	+1	31301.53	$a^4P'_1-a^4D'_1$
3196.086	3	+1	31279.25	$a^4P'_3-a^4F_4$
3210.458	4	+2	31139.24	$a^4P'_1-a^4D'_2$
3213.320	5	+1	31111.50	$a^4P'_2-a^4D'_3$
3227.756	7d	+1	30972.36	$a^4P'_3-a^4D'_4$
3255.898	2	0	30704.67	$a^4D_4-a^6D'_4$
3277.358	3	0	30503.62	$a^4D_4-a^6D'_5$
3281.302	2	0	30466.95	$a^4D_3-a^6D'_3$
3295.825	1	−1	30332.72	$a^4D_2-a^6D'_2$
3302.865	1	0	30268.06	$a^4D_3-a^6D'_4$
3303.476	1	0	30262.44	$a^4D_1-a^6D'_1$
3312.707	1	+4	30178.12	$a^4D_2-a^6D'_3$
3974.17	*	0	25155.38	$b^4P'_2-a^4P_3$?
4122.67	*	+1	24249.26	$b^4P'_3-a^4F_3$
4128.74	*	+1	24213.67	$b^4P'_3-a^4D'_2$
4173.475	6	+1	23954.12	$b^4P'_3-a^4D'_3$
4178.868	6	+1	23923.22	$b^4P'_3-a^4F_4$
4233.163	8	0	23616.36	$b^4P'_3-a^4D'_4$
4258.16	*	−1	23477.87	$b^4P'_2-a^4F_2$
4273.31	1	−1	23394.50	$b^4P'_2-a^4D'_1$
4296.56	6	−2	23267.93	$b^4P'_2-a^4F_3$
4303.18	4	+2	23232.10	$b^4P'_2-a^4D'_2$
4351.77	6	−1	22972.72	$b^4P'_2-a^4D'_3$
4369.41	*	+1	22880.01	$b^4P'_1-a^4F_2$
4385.39	*	+1	22796.60	$b^4P'_1-a^4D'_1$
4416.81	4	0	22634.42	$b^4P'_1-a^4D'_2$
4472.93	*	0	22350.56	$b^4F'_3-a^4F_2$
4489.21	4	+2	22269.38	$b^4F'_4-a^4F_3$
4491.41	4	+1	22258.48	$b^4F'_2-a^4F_2$

TABLE II—*Continued*

λ	Int.	O−C λ	ν Vac.	Multiplet
4508.287	8	− 1	22175.16	$b^4F'_2-a^4D'_1$
4515.337	6	0	22140.55	$b^4F'_3-a^4F_3$
4520.238	6	0	22116.55	$b^4F'_5-a^4F_4$
4522.636	6	+ 2	22104.82	$b^4F'_3-a^4D'_2$
4534.17	*	− 3	22048.66	$b^4F'_2-a^4F_3$
4541.52	*	− 2	22013.01	$b^4F'_2-a^4D'_2$
4549.477	4	− 2	21974.45	$b^4F'_4-a^4D'_3$
4555.904	6	0	21943.44	$b^4F'_4-a^4F_4$
4576.31	4	− 3	21845.56	$b^4F'_3-a^4D'_3$
4582.84	*	+ 3	21814.39	$b^4F'_3-a^4F_4$
4583.843	8	0	21809.68	$b^4F'_5-a^4D'_4$
4595.70	*	+ 1	21753.30	$b^4F'_2-a^4D'_3$
4601.38	*	0	21726.54	$a^6S-a^4D'_2$
4620.52	*	+ 1	21636.50	$b^4F'_4-a^4D'_4$
4629.327	4	0	21595.38	$b^4F'_5-a^4F_5$
4648.32	*	+10	21507.15	$b^4F'_3-a^4D'_4$
4656.98	*	− 3	21467.14	$a^6S-a^4D'_3$
4663.71	*	− 3	21436.23	$a^6S-a^4F_4$
4666.75	*	0	21422.22	$b^4F'_4-a^4F_5$
4731.488	1	+ 2	21129.11	$a^6S-a^4D'_4$
4923.92	10	0	20303.37	$a^6S-a^6P_2$
5018.437	8	− 1	19920.95	$a^6S-a^6P_3$
5169.029	8	+ 1	19340.60	$a^6S-a^6P_4$
5197.56	4	− 2	19234.47	$a^4G_3-a^4F_2$
5234.62	4	− 1	19098.29	$a^4G_4-a^4F_3$
5256.94	*	− 6	19017.22	$a^6S-a^6F_3$
5264.81	1	+ 1	18988.77	$a^4G_3-a^4D'_2$
5276.012	6	+ 2	18948.44	$a^4G_5-a^4F_4$
5284.11	*	− 1	18919.41	$a^6S-a^6F_4$
5316.62	8	0	18803.74	$a^4G_6-a^4F_5$
5316.78	*	− 2	18803.17	$a^4G_4-a^4D'_3$
5325.56	4	+ 2	18772.15	$a^4G_4-a^4F_4$
5346.54	*	0	18698.37	$a^4G_3-a^4F_4$
5362.87	*	+ 3	18641.49	$a^4G_5-a^4D'_4$
5425.26	*	− 2	18427.37	$a^4G_5-a^4F_5$
6147.85	6	+10	16261.35	$b^4D_2-a^4P_1$
6149.25	4	− 1	16257.65	$b^4D_1-a^4P_1$
6238.39	4	− 2	16025.36	$b^4D_2-a^4P_2$
6239.95	*	0	16021.34	$b^4D_1-a^4P_2$
6247.56	4	− 1	16001.84	$b^4D_3-a^4P_2$
6369.48	*	+ 8	15695.23	$a^6S-a^6D'_2$
6407.30	*	+ 4	15602.89	$b^4D_2-a^4P_3$
6416.92	2	− 1	15579.50	$b^4D_3-a^4P_3$
6432.69	*	− 7	15541.40	$a^6S-a^6D'_3$
6456.39	6	− 3	15484.26	$b^4D_4-a^4P_3$
6516.08	4	− 4	15342.41	$a^6S-a^6D'_4$
7222.415	*	+ 4	13842.00	$b^4D_2-a^4D'_1$
7224.481	*	+ 3	13838.03	$b^4D_1-a^4D'_1$
7301.587	000	+ 4	13691.88	$b^4D_3-a^4F_3$
7308.09	000	+ 8	13679.69	$b^4D_2-a^4D'_2$?
7310.226	00(A)	+10	13675.66	$b^4D_1-a^4D'_2$
7320.72§	0	+12	13656.10	$b^4D_3-a^4D'_2$
7449.36	*	+ 3	13420.26	$b^4D_2-a^4D'_3$
7462.356‡	2	− 9	13396.97	$b^4D_3-a^4D'_3$

TABLE II—*Continued*

λ	Int.	$\frac{O-C}{\lambda}$	ν Vac.	Multiplet
7515.89........	*	+ 1	13301.50	$b^4D_4 - a^4D'_3$
7533.418.......	*	+ 5	13270.53	$b^4D_4 - a^4F_4$
7655.474.......	ooNA	+ 3	13058.94	$b^4D_3 - a^4D'_4$
7711.759.......	*	+ 3	12963.66	$b^4D_4 - a^4D'_4$

* (In intensity column.) Not recorded in laboratory spectra, but visible as a very faint line in sun-spot spectra.

† Blend with Fe^+.

§ Blend with *Fe*.

|| Blend with arc line? Spark line should be at λ 3192.695 or mistake in identification. Z.E. is correct for spark line.

‡ May be masked by *Cr*.

spond not one, but many spectroscopic terms, which owe their differences presumably to different quantized orientations of the various orbits in space. What terms correspond to any given electronic configuration may be worked out on Hund's principles.

The singly ionized atom of iron contains seven electrons outside the complete "argon-shell." In the states of lowest energy these will be in 4s or 3d orbits—the state in which there are two of the former and five of the latter being represented for brevity by s^2d^5.

The spectroscopic terms resulting from these states, according to Hund's theory, are as follows:

	Sextets	Quartets	Doublets
s^2d^5...............	S	P′DF′G	SDF′GI, P′DF′GH′, D
sd^6...............	D	D, P′F′, P′DF′GH′	SDF′GI, SDG
d^7...............	....	P′F′	P′DF′GH′, D

According to Hund, among the terms arising from a given configuration, those of the highest multiplicity will be the lowest. The ^6D- and ^6S-terms may therefore be assigned with certainty to the configurations sd^6 and s^2d^5. The lowest $^4F'$- and $^4P'$-terms may be assigned with practically equal certainty to d^7—the principal ground for this being comparison with terms arising from similar configurations in the spark spectra of *V*, *Cr*, and *Mn*, which show that the configurations d^{n+1} and sd^n always give terms at nearly the same energy-level. The higher quartet P′DF′G-terms probably arise from sd^6.

None of these terms should, theoretically, combine with one another to give spectral lines, but all of them should combine, when the inner-quantum rules allow, with the terms arising from the configurations d^6p, etc. The combinations $^6S-{}^6D'$, $^6S-{}^6F$ are instances of this. The number of such terms is very great. The configuration d^6p alone accounts for the following:

Type	S′	P	D′	F	G′	H	I′	J
Sextets	....	1	1	1	....	....	....	....
Quartets	2	4	6	5	4	2	1	....
Doublets	2	7	8	9	7	5	2	1

The three observed sextet terms are of just the types predicted. Three of the quartet terms, which appear in most of the combinations, form a similar "triad" and are probably very closely related to the sextet terms, having as limits the same term, ^5D, in the *Fe* III spectrum. From what arrangement of orbits the other ^4F-term comes cannot yet be stated.

So many more terms are predicted, even by this one electronic configuration, that it is not surprising that there are many unclassified lines. The doublet system, which has not yet been unraveled, should be extremely intricate. It is practically certain that the levels predicted and not yet found lie higher up than those so far known, and give lines well out in the ultra-violet. A full analysis of this spectrum will probably demand complete lists of spark lines in the region from λ 2200 onward. The observed energy-levels are just those which might have been anticipated on the basis of Hund's theory as giving the most prominent lines. The only point of unusual interest is that three different electronic configurations, s^2d^5, sd^6, d^7, appear to contribute to the production of the low terms. In the spark spectra of the preceding elements of this period, so far as known to the writer, only the configurations sd^n and d^{n+1} produce such terms.

Mount Wilson Observatory
July 1926

REVIEWS

Stellar Atmospheres. A Contribution to the Study of High Temperature Ionization in the Reversing Layers of Stars. By CECILIA H. PAYNE. Cambridge, Mass., 1925. 8vo. Pp. ix+215. Figs. 10. $2.50.

Miss Payne's new book on *Stellar Atmospheres* is the first volume of a series of monographs designed to contain the results of special investigations made at the Harvard College Observatory. The monographs appear under the editorship of Director Harlow Shapley, and are published and distributed by Harvard Observatory. A special gift from Mrs. James R. Jewett has made it possible to sell the present volume at less than cost.

Stellar Atmospheres embraces the newest branch of astrophysics. A decade or so ago astronomical spectroscopy consisted almost exclusively of the study of the positions and displacements of spectral lines in various stars. The results derived from such studies were used for purposes of identification and for the determination of radial velocities. Modern theories of light have brought out the importance of determining the intensities of spectral lines. It is this study with which Miss Payne's book is chiefly concerned.

The book is divided into three parts. Part I, containing eighty-eight pages, is entitled "The Physical Groundwork." The first chapter of this part deals with the physical basis of astrophysics, and contains a short but comprehensive exposition of modern theories of spectra. Particularly interesting are pages 14–21 dealing with ionization and excitation potentials. A useful diagram of ionization potentials plotted against atomic numbers is given in Figure 4, page 20. The second chapter deals with the stellar temperature scale and refers to the work of Scheiner, Wilsing, E. S. King, Rosenberg, Abbot, Coblentz, and others. In chapter iii, which the author calls "Pressures in Stellar Atmospheres," particular attention is given to the work of H. N. Russell and of Stewart. Very interesting is the reference on page 42 to Miss Payne's own work concerning a possible correlation between absolute magnitude and the observed number of the Balmer lines of hydrogen, in A-type stars. The fourth chapter gives a general description of stellar spectra. The fifth discusses the various elements and compounds observed in the stars, and is one of the most important for the practical astrophysicist. The elements are arranged in order of their atomic numbers, and are complete as far as stellar occurrence is concerned.

Part II, which contains sixty-four pages (chaps. vi–x), deals with the theory of thermal ionization. Chapter vi discusses the production by thermal excitation of ultimate and subordinate lines of neutral and ionized atoms. Chapter vii explains the theoretical work of Saha, of Fowler and Milne, and others. The space allotted to this chapter is insufficient to make the mathematical part complete, and readers will find it necessary to refer to the original papers in various publications. Chapter viii contains a large amount of observational data derived by Miss Payne from the Harvard collections of stellar spectra. Table XIX (pp. 121–126) gives the estimated intensities of a large number of lines for stars of practically the whole sequence of stellar classification. The material collected in this table has only in part been utilized for theoretical purposes and may well serve as a source for further studies. Chapter ix deals with the temperature scale of ionization derived by Miss Payne. This scale is based, not on the marginal appearance of certain lines, as was originally suggested by Saha, but on the more easily observed maxima of the intensities. Chapter x discusses the effects of absolute magnitude upon the spectrum. Particularly interesting is a comparison of the observed effects with theoretical predictions.

Part III, which contains forty-eight pages (chaps. xi–xv), includes some additional deductions from the theory of ionization. Chapter xi is entitled "The Astrophysical Evaluation of Physical Constants." Chapter xii discusses certain special problems, such as the O-type stars and the abnormal A stars having strong silicon or strontium lines. Chapter xiii expounds Miss Payne's own work on the abundance of the elements, as derived from the observation of the first and last appearance of certain lines. Chapter xiv is devoted to a discussion of the meaning of the spectral classification, and the concluding chapter, xv, takes into consideration the future of the problem.

Five appendices are added to the book, giving an "Index to Definitions," "Series Relations in Line Spectra," "List of Stars Used in Table XIX," "Intensity Changes of Lines with Unknown Series Relations," and "Material on A Stars."

The book is supplied with indexes by authors and by subjects. The figures are not numerous but well chosen, illustrating chiefly the intensity curves of various lines. The book contains only one photographic reproduction from a Harvard plate, showing objective-prism spectrograms of various stars. The value of the book could undoubtedly be increased by adding a few more photographs of stellar spectra, preferably on a larger scale.

The material which constitutes the bulk of Miss Payne's book is so new that it can hardly be expected to represent the views of all investigators. In doubtful cases Miss Payne states her own opinions, leaving it to the reader to accept or reject them.

On page 58 the following statement is made: "The helium lines cannot . . . . be used in the estimation of spectroscopic parallaxes. The question of absolute magnitude effects cannot be usefully pursued in the absence of more reliable parallaxes, for the B-type stars, than are at present available." This statement seems entirely too radical and unjustified in view of the work of Adams and Joy at Mount Wilson, as well as that of Edwards in England, on the distances of B-type stars. It is true that certain difficulties exist in the determination of spectroscopic parallaxes of stars of early types. However, there are various possibilities which have not yet been applied to the problem. Shajn has suggested that double stars might be used for the calibration of certain spectroscopic phenomena, since in this case $\Delta M = \Delta m$. Furthermore, there are various moving clusters of B-type stars which have not been fully utilized. An important cluster of stars of classes B and O exists, according to W. S. Adams and A. van Maanen, and also according to J. S. Plaskett, in the constellation Perseus. Various other cases of group motion are evident in Lacerta and Orion, not to mention the group motion in the southern hemisphere discovered by Kapteyn, in the Pleiades, and in Eddington's cluster in Perseus. Thus it seems to me that there is a large amount of material that could be used for establishing true differences in luminosity, if not true absolute magnitudes.

On page 169 the statement is made that the "sharpness of the hydrogen lines has indeed been used at Mount Wilson as a quantitative measure of luminosity" in A stars. This statement seems to be not quite correct as the designation of "n" and "s" in the Mount Wilson system is based on all lines, not on those of hydrogen alone. As a matter of fact, the high dispersion obtained with ordinary slit spectrographs makes the hydrogen lines unsuitable for such an investigation. In my own work I have found the lines *Mg* 4481 and *Ti* 4549 especially suitable for the determination of line-widths.

It is regrettable that Miss Payne has not adopted H. H. Plaskett's scheme for the classification of O stars. The original Harvard classification is not quite satisfactory at the two ends of the sequence of spectra. Such designations as Oe5 or Oe are not only meaningless with regard to stellar evolution, but they are even confusing, since the letter "e" is generally accepted to mean presence of emission lines. Thus Oe on the Inter-

national System would indicate any O star having emission lines, without giving it a definite place within the group of O stars. Miss Payne does not adhere to the Harvard system, although she uses it to some extent. Her sequence of individual stars, while comprehensible to the practical worker, will be entirely meaningless to anyone not experienced in classifying stellar spectra. In a former publication[1] Miss Payne has raised certain definite objections to the scheme of H. H. Plaskett, and has suggested that we abandon the classification of O stars altogether and arrange the individual stars in an evolutionary sequence. In spite of these difficulties, a uniform classification from O5 to O9 would be much more convenient.

On page 168 Miss Payne speaks of the classification of A stars as adopted in the *Henry Draper Catalogue* and remarks that "the problem of their classification is one of the future tasks of astrophysics." This point should perhaps have been made even more emphatic. It is a frequent occurrence that stars belonging to the same spectral subdivision of type A have widely different spectra. There are AO stars which have no lines of titanium whatever, and there are other stars of the same subdivisions which in their appearance resemble type F. Evidently the Harvard classification, which can be termed as "linear," is insufficient at this point, and a more complex system is greatly needed. It is for this reason that there exist differences in opinion as to the validity of the attributes "n" and "s" in determining absolute magnitudes. If the increasing intensity of the titanium lines is used as a basis of classification, there exists a marked correlation between line-width and absolute magnitude. This correlation breaks down almost completely as soon as the Harvard criteria are used in classifying the spectra. Hence the difference in the results of Miss Fairfield and those of Adams and Joy, referred to on page 57. However, the spectra of physically connected double stars seem to indicate that, if the two spectra are similar, the lines are wider in the fainter components.

The reference on page 71 to stationary *Ca* lines is far from complete. The reader is led to believe that the ordinary stellar lines H and K are seen throughout the stellar sequence. This is not the case, for they disappear at or near type B3 and are not definitely known to occur in the earlier types. The stationary lines have probably no direct connection with stellar atmospheres.

In Table V on page 30 Rosenberg's value for AO should evidently read "12,000°" instead of "1200°."

[1] *Harvard College Observatory Circular*, No. 263.

Miss Payne's book is full of useful suggestions for the practical worker. Nearly every page contains references to problems which are open to investigation by the spectroscopist. The behavior of sulphur in B stars and of oxygen in A stars, the peculiarities of the strontium and silicon lines, effects of absolute magnitude, and abnormal intensities of various lines provide ample material for many years of work. The pure physicist will find in this volume a unique exposition of the results derived under conditions of extreme heat.

Stellar Atmospheres while written in a popular and interesting style, differs from the conventional treatise on astronomy in that it is intended not so much for the layman as for the advanced student and the specialist. It is an important addition to American bibliography as a worthy successor to such standard works on astrophysics as Frost's revision of Scheiner's *Astronomical Spectroscopy* and Campbell's *Stellar Motions*.

OTTO STRUVE

A Graphic Table Combining Logarithms and Anti-Logarithms. By ADRIEN LACROIX and CHARLES L. RAGOT. New York: Macmillan Co., 1925. 8vo. Pp. 52. $1.40.

The book is in two parts. The first forty pages contain the logarithms and anti-logarithms for five places of decimals, while the second part, or four-place table, is compressed into six pages.

The graphic scale of logarithms is paralleled by one of anti-logarithms and enables one to read off directly to five places of decimals the quantity sought, without interpolation; and the scale is so large that it is easy to read. The diagrams are compact, thus saving much time in finding the numbers required.

This should prove a boon to many who do not now use logarithms because of the necessary interpolations, and should stimulate the use of logarithms in many computations which are now carried out by other means.

The Introduction lacks any explanation of the theory of logarithms. The book would be more convenient if it included the logarithms of trigonometric functions.

E. M. JUSTIN